N. KUROKAWA AND H. TANAKA
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ABSOLUTE ZETA FUNCTIONS AND THE AUTOMORPHY

Nobushige Kurokawa and Hidekazu Tanaka

Introduction

In this paper we study the absolute zeta function $\zeta_f(s)$ associated to a certain "absolute automorphic form" $f(x)$ on the group

$$\Gamma = \mathbf{R}_{>0} = \{x \in \mathbf{R} \mid x > 0\}.$$

We require $f(x)$ to satisfy the following automorphy:

$$f\left(\frac{1}{x}\right) = Cx^{-D}f(x),$$

where $D \in \mathbf{Z}$ with $C = \pm 1$.

To explain our problem we first recall the history of absolute zeta functions briefly. Soulé [17] (2004) introduced the absolute zeta function (the zeta functions over $\mathbf{F}_1$) of a suitable scheme X as the limit of the congruence zeta function

$$\zeta_{X/\mathbf{F}_1} = \lim_{p \to 1} \zeta_{X/\mathbf{F}_p}(s),$$

where

$$\zeta_{X/\mathbf{F}_p}(s) = \exp\left(\sum_{m=1}^{\infty} \frac{|X(\mathbf{F}_{p^m})|}{m} p^{-ms}\right);$$

see Kurokawa [12] (2005) and Deitmar [4] (2006). Later Connes-Consani [2] (2010) [3] (2011) interpreted it as

$$\zeta_{X/\mathbf{F}_1}(s) = \exp\left(\int_1^{\infty} \frac{f(x)x^{-s-1}}{\log x}\,dx\right)$$

when

$$f(x) = |X(\mathbf{F}_x)| \in \mathbf{Z}[x].$$

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