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METRICS AND CONNECTIONS ON THE COTANGENT BUNDLE

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§1. Introduction.

Let M be an n-dimensional differentiable manifold of class C^{∞} and T^*M its cotangent bundle, which is a $2n$-dimensional differentiable manifold. The problem of extending structures on M to T^*M has been the subject of a number of papers. An account of these can be found in Yano and Ishihara [12]. Starting from a torsion-free linear connection on M, Patterson and Walker [5] have shown how to construct a metric on T^*M, a process which they called the *Riemann extension*. Using the Riemann extension, Yano and Patterson [13, 14] have defined the *complete* and *horizontal lifts* of linear connections on M to T^*M. On the other hand, Tondeur [8] and Sato [7] have constructed a metric on T^*M from a metric on M, the construction being the analogue of the metric of Sasaki for the tangent bundle TM [6].

When a linear connection is given on M, we may view T^*M as an almost product manifold. Linear connections on an almost product manifold have been studied by Walker [9], Yano [10] and Davies [2], among others. In the present paper, we shall further consider the metrics and connections on T^*M mentioned above, bringing in the general theory of linear connections on an almost product manifold whenever possible. In this way, the relations between the various metrics and connections become clearer, and we obtain a new linear connection on T^*M, namely the *intermediate lift*, which, in some sense, lies somewhere between the complete and horizontal lift. Refering to the "adapted frames" on T^*M, we have computed the components of the curvature tensors of the various linear connections on T^*M, as well as that of their covariant derivatives.

Similar considerations to the metrics and connections on the tangent bundle TM can be found in the papers of Davies [2] and Yano and Davies [11]. In fact, our intermediate lift is the cotangent bundle analogue of the connection $\mathring{\nabla}$ on TM appearing in [2, §4].

As to notations and definitions, we shall generally follow that in [12]. In particular:

1) Indices $a, b, c, \cdots ; h, i, j, \cdots$ have range in $\{1, \cdots, n\}$, while indices $A, B, C, \cdots ; \lambda, \mu, \nu, \cdots$ have range in $\{1, \cdots, n ; n+1, \cdots, 2n\}$. We put $\bar{i}=n+i$. Summation over repeated indices is always implied. Entries of matrices are written

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