K. NISHIOKA
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ON THE STABILITY OF TWO-DIMENSIONAL LINEAR STOCHATIC SYSTEMS

BY KUNIO NISHIOKA

1. Introduction and preliminaries.

Consider a two-dimensional linear system of temporally homogeneous stochastic differential equations:

$$dX(t)=B\cdot X(t)dt+C\cdot X(t)dB_1(t)+D\cdot X(t)dB_2(t) \tag{1.1}$$

where B, C, and D are 2×2 constant matrices and $B_i(t)$ $(i=1, 2)$ are independent Brownian motions. Our concern is the asymptotic stability with probability 1 of the system (1.1), i.e., we say that $X^{x_0}(t)$ is stable if

$$P_{x_0}\{\lim_{t\to\infty}|X(t)|=0\}=1,$$

and that it is divergent if

$$P_{x_0}\{\lim_{t\to\infty}|X(t)|=\infty\}=1$$

(here and later on $X^{x_0}(t)$ stands for a solution of (1.1) satisfying $X^{x_0}(0)=x_0$). Applying Ito's formula to $\rho(t)\equiv\log|X(t)|$, Khas'minskii [6] showed that

$$\lim_{T\to\infty}\frac{1}{T}(\rho(T)-\rho(0))=\lim_{T\to\infty}\frac{1}{T}\int_0^T Q(\theta(t))dt \qquad \text{a. s.,} \tag{1.2}$$

where $\theta(t)$ is the angular component of $X(t)$ and

$$Q(\theta)\equiv(B\cdot e(\theta), e(\theta))+\frac{1}{2}\mathrm{Sp}\cdot A(e(\theta))-(A(e(\theta))\cdot e(\theta), e(\theta)), \tag{1.3}$$

in which

$$a(x)_{ij}\equiv\sum_{m,n=1}^{2}(c_{im}c_{jn}+d_{im}d_{jn})x_m x_n \tag{1.4}$$

$$e(\theta)\equiv(\cos\theta, \sin\theta)$$

(we denote by c_{ij} and x_i an (i, j)-element of a matrix C and an i-element of a vector X, respectively, etc.). Then, he has proved: if

$$J\equiv\lim_{T\to\infty}\frac{1}{T}\int_0^T Q(\theta(t))dt$$

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