K. TAKAMATSU AND T. SATO
KŌDAI MATH. SEM. REP.
27 (1976), 116—127

A K-SPACE OF CONSTANT HOLOMORPHIC SECTIONAL CURVATURE

By Kichiro Takamatsu and Takuji Sato

In previous paper [6], Y. Watanabe and one of the present authors proved that a K-space of constant holomorphic sectional curvature is an Einstein space. The purpose of the present paper is to prove the following

THEOREM. *There does not exist any dimensional, except 6 dimensional, non-Kähler K-space of constant holomorphic sectional curvature.*

In this paper, by a K-space we shall mean a non-Kähler K-space. In § 1, we shall give preliminary facts. In § 2, we shall prepare some lemmas which are useful for proof of the theorem. The last section will be devoted to proof of the theorem.

1. Preliminaries. Let M be an n-dimensional $(n>2)$ almost Hermitian manifold with Hermitian structure $(F_j{}^i, g_{ji})$. If this structure satisfies

$$\nabla_j F_{ih}+\nabla_i F_{jh}=0\,, \tag{1.1}$$

where ∇_j denotes the operator of covariant differentiation with respect to the Riemannian connection, then the manifold is called a *K-space* (or *Tachibana space*, or *nearly Kähler manifold*).

Now, in a K-space let $R_{kji}{}^h$, $R_{ji}=R_{tji}{}^t$ and $R=g^{ji}R_{ji}$ be the Riemannian curvature tensor, the Ricci tensor and the scalar curvature respectively. Moreover we put

$$R^*{}_{ji}=\frac{1}{2}F^{ba}R_{bati}F_j{}^t\,, \qquad R^*=g^{ji}R^*{}_{ji}\,.$$

Then we have the following identities [3], [4]:

$$\nabla_j F_{ih}+F_j{}^b F_i{}^a \nabla_b F_{ah}=0\,, \tag{1.2}$$

$$R_{ji}=F_j{}^b F_i{}^a R_{ba}\,, \qquad R^*{}_{ji}=F_j{}^b F_i{}^a R^*{}_{ba}\,, \tag{1.3}$$

$$R^*{}_{ji}=R^*{}_{ij}\,, \tag{1.4}$$

$$\nabla_j F_{ts}(\nabla_i F^{ts})=R_{ji}-R^*{}_{ji}\,, \tag{1.5}$$

Received Feb. 22, 1974.