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A NOTE ON SEMIGROUPS OF MARKOV OPERATORS ON $C(X)$

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1. Introduction.

Let X be a compact Hausdorff space, and let $C(X)$ be the commutative C^*-algebra of all continuous complex functions on X. A bounded linear operator T of $C(X)$ into itself is called a Markov operator it $T\geqq 0$, $\|T\|=1$, and $T1=1$.

Let Σ be a semigroup of Markov operators. For each $f\in C(X)$, $\overline{\text{co}}\{Tf: T\in\Sigma\}$ denotes the closed convex hull of $\{Tf: T\in\Sigma\}$. $g\in C(X)$ is called a Σ-invariant function if $Tg=g$ for all $T\in\Sigma$.

In ergodic theory the following conditions on Σ are interesting: (I) Each $\overline{\text{co}}\{Tf: T\in\Sigma\}$ contains exactly one Σ-invariant function. (II) Each $\overline{\text{co}}\{Tf: T\in\Sigma\}$ contains at least one Σ-invariant function. In Theorem 1, we shall give some necessary and sufficient conditions that (I) holds.

Let $C(X)^*$ be the dual Banach space of $C(X)$. $\mu\in C(X)^*$ is called a state if $\mu\geqq 0$ and $\|\mu\|=\mu(1)=1$. If T is a Markov operator and if μ is a state, then $T^*\mu$ is also a state where T^* denotes the adjoint operator of T. A state μ is called a Σ-invariant state if $T^*\mu=\mu$ for all $T\in\Sigma$.

Let K_Σ be the set of all Σ-invariant states. Then K_Σ is a weak*-compact convex subset of $C(X)^*$. $\mu\in K_\Sigma$ is called an extremal Σ-invariant state if μ is an extreme point of K_Σ.

A proper closed ideal I of $C(X)$ is called a Σ-invariant ideal if $T(I)\subset I$ for all $T\in\Sigma$. There exists at least one maximal Σ-invariant ideal, and each Σ-invariant ideal is contained in some maximal Σ-invariant ideal. If μ is a Σ-invariant state, then $I_\mu=\{f\in C(X): \mu(|f|)=0\}$ is a Σ-invariant ideal.

In Theorem 2, we shall show that if (I) holds, then $\mu\to I_\mu$ is a bijection of the set of all extremal Σ-invariant states onto the family of all maximal Σ-invariant ideals.

Our discussion is much due to Deleeuw and Glicksberg [1], Schaefer [2], Sine [3], and Takahashi [4].

2. Theorems.

co Σ denotes the set of all finite convex linear combinations of operators in Σ. co Σ is also a semigroup of Markov operators. We note that $\overline{\text{co}}$ $\{Tf:$

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