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KŌDAI MATH. SEM. REP.
25 (1973), 104—110

A NOTE ON ANALYTIC SELF-MAPPINGS

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Dedicated to Professor Yukinari Tôki on his 60th birthday

1. Statements and notations. Recently, Marden, Richards and Rodin [7] studied analytic self-mappings of Riemann surfaces and proved a number of theorems related to homotopy some of which involve Huber's results [4]. Jenkins and Suita [5] also investigated this theory from the other point of view. In their paper they gave an alternative proof of the result in [7]. As a counterpart of it they proved a similar result related to homology only for plane regions. In the present paper we shall extend their result to Riemann surfaces.

Throughout this paper if nothing otherwise is indicated we denote by W a Riemann surface whose fundamental group is non-abelian, and by $\hat{W}$ the Kerékjártó-Stoïlow compactification of W. $H_1(W)$ denotes the 1-dimensional homology group of W. Let f be an analytic self-mapping of W. It is known that f induces an endomorphism E_f by the mapping $c \to f(c)$ for $c \in H_1(W)$. Let f^n denote the n-th iteration of f. Then our main theorem is stated as follows:

THEOREM. *Let H be a non-trivial subgroup of $H_1(W)$ with finite rank. Suppose that the restriction of the induced homomorphism $E_f|H$ is an endomorphism. If the kernel of the restriction $E_f|H$ is trivial, f is either an automorphism of finite period or $\{f^n\}$ tends to an isolated ideal boundary component of harmonic dimension one uniformly on every compact subset of W. In the latter case H reduces to an infinite cyclic group generated by a dividing cycle. Furthermore, if the ideal boundary component is a non-planar boundary then $E_f|H$ reduces to the identity mapping.*

This theorem is also an extension of a theorem of Komatu and Mori [6]. The authors express their heartiest thanks to Professor N. Suita for his encouragement in preparing this paper.

2. Lemmas. It is convenient to give some preparatory lemmas. The following lemma related to the iteration of analytic self-mappings is found in Heins [3].

LEMMA 1. *Suppose that K_1 and K_2 are given compact subsets of W. If f neither possesses a fixed point nor has finite period, then $f^n(K_1)$ lies in one and the same component of $W-K_2$ for n sufficiently large. If f has a fixed point and is*

Received April 25, 1972.