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A LIMIT PROPERTY OF SEQUENTIAL DECISION PROCESS

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1. Introduction.

In this paper we consider a limit property of sequential design problem. Generally, in the sequential design problem, the following subjects are important for us. (1) Which experiment we must select at each step? (2) When we must stop experiment? Bradt, Johnson and Karlin [1] have given a famous example of sequential design problem, named as "Two Armed Bandit Problem". In the sequential testing model of composite hypothesis, Chernoff [2] has given an asymptotically optimal procedure and its asymptotic behavior.

In this paper we shall give a set of finitely many experiments which are conditioned by the restrictions 1, 2, ⋯, 5 in the following section, and give a procedure $\mathcal{P}$ regarding only the subject (1). The process of gains given by the selected experiments being denoted as $\{X_n\}$ under the procedure $\mathcal{P}$, it will be shown that the sample mean $\overline{X}_n=\sum_{i=1}^{n}X_i/n$ given at n-th step has a maximum limit value as n tends to infinity. Under another procedure $\mathcal{P}'$, regarding only the subject (1), the process $\overline{X}_n$ has not always a limit value, but, if the process $\overline{X}_n$ converges under $\mathcal{P}'$, the limit value will be not greater than the limit value under the procedure $\mathcal{P}$.

2. Notations, restrictions of experiments and definition of procedure.

In this paper we treat a set of k mutually independent experiments (trials) $E_1, \cdots, E_k$. The chance variables $X_{E_1}, \cdots, X_{E_k}$ of the trials have unknown mean values $m_1, \cdots, m_k$ respectively. We assume that we have not any information for $m_1, \cdots, m_k$ until we observe the first result of the trials E_1 or E_2 or $\cdots$ or E_k. In the first step of the selection we are admitted to select each one of the given k trials $E_1, \cdots, E_k$. In the second step we are admitted to select the second trial investigating our own purpose under given informations of the result of first step, and so on. And we assume that the result of a fixed step was independently distributed to the preceding selections of trials. Our purpose of selections is to maximize asymptotically the process of the sample mean $\overline{X}_n$ given by the first n observations $X_1, \cdots, X_n$ of the first n selections of trials $E^{(1)}, \cdots, E^{(n)}$. Here we denote the first n observations as $X_1, \cdots, X_n$, and especially we shall not write the trial suffix in the following lines. We shall decide the selecting way of trial for each step

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