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CARATHÉODORY'S THEOREM ON BOUNDARY ELEMENTS OF AN ARBITRARY PLANE REGION

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1. Introduction. Let $\varDelta$ be a simply connected hyperbolic region. By defining prime ends on the boundary of $\varDelta$ we can get a compactification of $\varDelta$, denoted by $\tilde{\varDelta}$ and every conformal mapping of $\varDelta$ is extended to a topological mapping of $\tilde{\varDelta}$ onto the compactification of the image domain [2]. Especially, if $\varDelta$ is the interior of a Jordan curve, called a Jordan region, the realization (impression) of every prime end is a point on the curve and conversely every point on the curve represents a unique prime end. This is termed Carathéodory's theorem [1].

The purpose of the present note is to generalize this theorem to boundary elements of an arbitrary region. They were introduced as a generalization of a prime end by the author [6]. We shall define an almost Jordan region in terms of extremal length and show that the realizations of boundary elements are mutually disjoint continuums, each of which has a nonvoid connected intersection with its defining Jordan curve. Furthermore, we shall define a weak boundary element and prove that its realization on an almost Jordan region is always a point. The terminology of a weak boundary component was used by Sario [5] which was first investigated by Grötzsch [3]. A weak boundary element is related to a boundary point with vanishing extremal diameter introduced by Strebel [6] when the realization of the boundary component containing the point is a Jordan curve.

2. Extremal length. Let Ω be an arbitrary plane region and let Γ be a family of locally rectifiable curves in Ω. Let $P(\Gamma)$ denote the class of nonnegative metrics $\rho(z)|dz|$ such that

$$\int_\gamma \rho(z)|dz| \geqq 1, \qquad \gamma \in \Gamma$$

which is called an admissible class of metrics. The module of Γ is defined by

$$\operatorname{mod} \Gamma = \inf_{\rho \in P(\Gamma)} \iint_\Omega \rho^2 dxdy$$

and its reciprocal is called the extremal length of Γ, denoted by $\lambda(\Gamma)$. If a curve family consists of the curves joining two sets, its extremal length is termed the extremal distance between these sets. A family of curves with vanishing module is said to be *exceptional*.

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