KÔDAI MATH. SEM. REP.
21 (1969), 133—150

MEASURE-THEORETIC CONSTRUCTION FOR INFORMATION THEORY

By Yatsuka Nakamura

1. Introduction.

In the measure theoretic viewpoints, the information theory originated by Shannon [13] can be divided into a couple of basic parts, that is, the one is concerned to information source and the other is concerned to information channel. Kolmogorov [10a, b] and Sinai [14] gave the concept of entropy of measure preserving transformation, modifying the method of information source, and they classified certain dynamical systems which belong to the same spectral type.

Halmos introduced a measure theoretic construction of information source in his lecture note [7]. Under such a measure theoretic form, we can apply the theory to both the classifications of dynamical systems and the composition of the concrete information theory constructed on alphabet spaces. In this paper, we shall study a measure theoretic construction of information channel. For this purpose, main themes are devoted to define channels, between two abstract measurable spaces, and ergodic or stationary capacities of such channels, and to find conditions under which these two capacities coincide.

At first, an integral representation of entropy function will be done for the latter intention, namely to find the conditions for coincidence of the capacities. Parthasarathy [12] and Jacobs [9a] proved that the representation is possible when entropy is defined on alphabet space, and Umegaki [17a] showed that it is also possible even when the space is a compact totally disconnected topological space. Their constructions are available for the case of the abstract dynamical system, reducing to the special cases by certain mappings (see [9b]). But the method employed here needs only some simple calculations of entropy, and some knowledges of the ergodic theorem and the martingale convergence theorem.

Seconderly necessary and sufficient conditions for ergodicity of channels will be researched. Hinchin believed in his paper [8] that finite memory channels are ergodic, who gave the first mathematical and systematical construction to discrete information theory originated by Shannon. But, Takano [15] pointed out that finite memory channels are not always ergodic and it needs a concept so called "M-dependence", in addition to the assumption of finite memory, for ergodicity of channels.

Adler [1] showed that "weakly mixing" and "strongly mixing" channels in

Received July 23, 1968.