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CORRECTION TO THE PAPER

"APPLICATION OF THE THEORY OF MARKOV PROCESSES TO COMMINUTION I. THE CASE OF DISCRETE TIME PARAMETER"

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The statement and proof of Lemma 2 on page 183 is incorrect. It should be replaced by the following

LEMMA 2′. *Let* $\{\zeta_n\}$ *be a sequence of nonnegative random variables and* $\{w_n\}$ *a sequence of positive numbers, and suppose that*

$$\frac{\zeta_n}{w_n} \to 1 \qquad \textit{in probability} \tag{5.19′}$$

as $n\to\infty$. *It is further assumed that for an integer* $k\geqq 1$ *the* k*-th moment* $E\{\zeta_n{}^k\}$ *exists and*

$$\lim_{n\to\infty} E\left\{\left(\frac{\zeta_n}{w_n}-1\right)^k\right\}=0. \tag{5.20′}$$

Then for all l *with* $0<l\leqq k$

$$\lim_{n\to\infty} \frac{E\{\zeta_n{}^l\}}{w_n{}^l}=1. \tag{5.21′}$$

Proof. We begin by showing that under the conditions of this lemma

$$\lim_{n\to\infty} E\left\{\left|\frac{\zeta_n}{w_n}-1\right|^k\right\}=0 \tag{5.22′}$$

holds. Now put $u_n=(\zeta_n/w_n-1)^k\geqq -1$ and $|u_n|=u_n^+ + u_n^- = u_n+2u_n^-$, where u_n^+ and u_n^- are the positive and negative parts of u_n, respectively. Then we have $0\leqq u_n^-\leqq 1$, so that (5.19′) implies $E\{u_n^-\}\to 0$. Hence $E\{|u_n|\}=E\{u_n\}+2E\{u_n^-\}\to 0$, as was to be proved.

It is easy to derive (5.21′) from (5.19′) and (5.22′). By Minkowski's inequality,

$$\left[E\left\{\left(\frac{\zeta_n}{w_n}\right)^l\right\}\right]^{1/l} \leqq \left[E\left\{\left(\frac{\zeta_n}{w_n}\right)^k\right\}\right]^{1/k} \leqq 1+\left[E\left\{\left|\frac{\zeta_n}{w_n}-1\right|^k\right\}\right]^{1/k},$$

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