TANGENT BUNDLE OF A MANIFOLD WITH A NON-LINEAR CONNECTION

By Ayako Kandatu

The concept of a non-linear connection was introduced by Friesecke, and was later studied by Kawaguchi and others [1, 2, 3, 4, 5, 6]. On the other hand, the geometry of tangent bundle of a Riemannian manifold has been studied by Sasaki and that of a Finslerian manifold by Yano and Davies [8, 9, 12].

In this paper, we shall study the geometry of the tangent bundle of a manifold with a non-linear connection. As is well known, a linear connection is by definition a mapping of $\mathfrak{X}\times\mathfrak{X}$ into $\mathfrak{X}$. Then, in § 1 we define a non-linear connection as a mapping ∇ of $\mathfrak{X}\times\mathfrak{X}$ into $\mathfrak{X}$, where $\mathfrak{X}$ is the totality of differentiable vector fields on the manifold. By studying vector fields on the tangent bundle, we shall show in § 2 that there exists an almost complex structure in the tangent bundle of a manifold with a non-linear connection. In § 3 we introduce the so-called adopted frame which is very useful for our discussions. § 4 is devoted to the study of integrability conditions of a non-linear connection and of the almost complex structure determined by a non-linear connection. Since the tangent bundle of a manifold with a non-linear connection admits an almost complex structure, we can define almost analytic vector fields on tangent bundle, which will be discussed in § 5.

§1. Non-linear connection.

Let $\mathfrak{F}(M^n)$ be the set of all differentiable functions of class C^∞ on an n-dimensional differentiable manifold M^n of class C^∞ and $\mathfrak{X}(M^n)$ the set of all differentiable vector fields of class C^∞ on M^n.

Let us suppose that there is given a mapping ∇: $\mathfrak{X}(M^n)\times\mathfrak{X}(M^n)\rightarrow\mathfrak{X}(M^n)$ satisfying the conditions:[1)]

$$
\begin{aligned}
&\text{(a)}\quad \nabla_{Y+Z}X=\nabla_Y X+\nabla_Z X,\\
&\text{(b)}\quad \nabla_{fY}X=f\nabla_Y X,\\
(1.1)\qquad &\text{(c)}\quad \nabla_Y(fX)=(Yf)X+f\nabla_Y X,\\
&\text{(d)}\quad (\nabla_Y X)_p=(\mathring{\nabla}_Y X)_p, \qquad \text{if}\quad X_p=0,\\
&\text{(e)}\quad (\nabla_Y(X+\bar{X}))_p=(\nabla_Y X)_p+(\nabla_Y\bar{X})_p, \qquad \text{if}\quad X_p+\bar{X}_p=0,
\end{aligned}
$$

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1) This definition was suggested by Professor S. Ishihara.