ON AN INVARIANT TENSOR UNDER A *CL*-TRANSFORMATION

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Tashiro and Tachibana showed some characteristic properties of Fubinian and C-Fubinian manifolds in their paper [6], where the notion of C-loxodromes was introduced in an almost contact manifold with affine connection.

The purpose of the present paper is to obtain an invariant tensor, that is, a tensor which is left invariant under a CL-transformation between two almost contact manifolds with symmetric affine connections. And Takamatsu and Mizusawa have performed the similar consideration about infinitesimal CL-transformations. [2].

§1. **Preliminaries.** [4, 5, 7, 8].

Let there be given, in an N-dimensional differentiable manifold M of class C^∞, a non-null tensor field f of type (1, 1) and of class C^∞ satisfying $f^3+f=0$. When the rank of f is constant everywhere and is equal to r, such a structure is called an f-structure of rank r. r is necessarily even.

Now, let M be a $(2n+1)$-dimensional differentiable manifold of class C^∞ for which the second axiom of countability holds true. If there exist a mixed tensor $f_j{}^i$, a contravariant vector field f^i and a covariant vector field f_j, all of which are of class C^∞, satisfying the conditions:

$$f^if_i=1, \qquad f_j{}^if_k{}^j=-\delta_k^i+f^if_k,$$

then such a manifold M is said to have an almost contact structure $(f_j{}^i, f^i, f_j)$ of class C^∞ and we call the manifold an almost contact manifold of class C^∞.

It is well-known that in a manifold with an almost contact structure $(f_j{}^i, f^i, f_j)$ of class C^∞, there exists a positive definite Riemannian metric g_{ji}, which is called a Riemannian metric associated with the almost contact structure, such that

$$f_i=g_{ij}f^j, \qquad g_{ji}f_h{}^jf_k{}^i=g_{hk}-f_hf_k.$$

We call the set $(f_j{}^i, f^i, f_j, g_{ji})$ an almost contact metric structure and a manifold with an almost contact metric structure $(f_j{}^i, f^i, f_j, g_{ji})$ of class C^∞ is called an almost contact metric (or Riemannian) manifold of class C^∞.

In a $(2n+1)$-dimensional differentiable manifold with an almost contact structure $(f_j{}^i, f^i, f_j)$, the following properties are satisfied:

$$(1.1) \qquad f^if_i=1,$$

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