ON FIBERINGS OF ALMOST CONTACT MANIFOLDS

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Introduction.

We study, in this paper, relations between almost contact structures and the induced complex structures. Throughout this paper, we consider the case in which an almost contact manifold $\tilde{M}$ is the bundle space of a principal fibre bundle over an almost complex manifold M.

We induce, in §1, an almost complex structure on M.

We consider, in §2, two sorts of torsions $\tilde{N}$ and $\tilde{\Phi}$ of an almost contact structure defined by Nijenhuis [4] and Sasaki-Hatakeyama [8] respectively, and investigate relations between the integrability of the induced almost complex structure on M and the vanishing of $\tilde{N}$ or $\tilde{\Phi}$.

In §3 we consider an almost contact metric structure on $\tilde{M}$ and induce an almost Hermitian structure on M and investigate relations between them.

In §4 we study, as special cases, an almost Sasakian structure and a Sasakian structure on $\tilde{M}$ and induce on M an almost Kähler structure and a Kähler structure.

In the last section we study a relation between the Riemannian connection on $\tilde{M}$ and on M and a relation between the curvature of $\tilde{M}$ and the curvature of M.

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§1. Regular almost contact structures.

Let $\tilde{M}$ be a $(2n+1)$-dimensional differentiable manifold. We denote by $\tilde{\mathfrak{X}}$ the Lie algebra of all vector fields on $\tilde{M}$.

An almost contact structure on $\tilde{M}$ is defined by a (1,1)-tensor field ϕ, a vector field ξ and a 1-form η satisfying the following conditions [7]:

$$\phi(\xi)=0, \tag{1.1}$$

$$\eta(\phi(\tilde{X}))=0 \qquad \text{for all} \quad \tilde{X}\in\tilde{\mathfrak{X}}, \tag{1.2}$$

$$\eta(\xi)=1, \tag{1.3}$$

$$\phi^2(\tilde{X})=-\tilde{X}+\eta(\tilde{X})\cdot\xi \qquad \text{for all} \quad \tilde{X}\in\tilde{\mathfrak{X}}. \tag{1.4}$$

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