GENERAL TREATMENT OF ALPHABET-MESSAGE SPACE AND INTEGRAL REPRESENTATION OF ENTROPY

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1. Introduction.

In this paper we shall clarify a topological structure of the alphabet-message space of the memory channel in information theory, and study the integral representation of entropy amount from a general view point of a certain generalized message space. In order to apply to the general theory of entropy, the present fashion will develop a message space into more general treatment, in which the basic space X will be assumed to be totally disconnected. As will be shown in §2, the alphabet-message space A^I is a totally disconnected compact space, and in §3, a kind of theorem relative to sufficiency for a σ-field generated by a partition and a homeoporphism (cf. Theorem 2) and the others (Theorems 3 and 4) are concerned with the semi-continuity of entropy amount which are general form of Breiman's Theorem [1]. Finally, in §4, it will be discussed about the function $h(x)$ found by Parthasarathy [7] whose integral defines the corresponding amount of entropy (cf. Theorem 5). It is also shown that the results in [9] can be generalized (cf. the footnote 2) below). The function $h(x)$ may give useful and interesting tool for the general theory of entropy of measure preserving automorphism or flow over a probabiltiy space.

2. Structure of message space.

A Hausdorff space X is called *totally disconnected* if X has a base consisting of closed-open sets, *clopen* say. In such a space X, a measure μ is called *normal*, if μ is regular and the mass of every non-dense set is zero. The space X is called *hyper-Stonean*, if it is compact and the union of carriers of all finite normal measures is dense in X. Such a space X is characterized by the existence of a normal measure μ (not necessarily finite) on X such that $\mu(G)>0$ for every non-empty open set G (cf. Dixmier [2]). Whence, the Banach space $C(X)$ of real continuous functions on X, with sup-norm $||\cdot||$, is isometrical and lattice isomorphic to the conjugate space of L^1-space $L^1(X, \mu)$. It is known that, these concepts on X are closely related with the theories of Boolean algebras and especially of operator algebras (=von Neumann algebras, cf. Dixmier [3]).

Received September 4, 1963.