LENGTH OF THE SINGULAR SET OF SCHOTTKY GROUP

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1. Let B_0 be an infinite domain on the z-plane, whose boundary consists of $2p$ ($p\geqq 2$) mutually disjoint circles H_i, H_i' ($i=1, 2, \cdots, p$). These circles are equivalent in pairs (H_i, H_i') ($i=1, 2, \cdots, p$); the outside of H_i is mapped onto the inside of H_i' by hyperbolic or loxodromic transformations

$$S_i: \quad z'=\frac{\alpha_i z+\beta_i}{\gamma_i z+\delta_i} \qquad (\alpha_i\delta_i-\beta_i\gamma_i=1).$$

The transformations S_i ($i=1, 2, \cdots, p$) generate a Schottky group G with the fundamental domain B_0.

2. Now let us define the grade of a transformation $S\in G$. Any element S of G is represented as the product of generators S_i ($i=1, 2, \cdots, p$) in the form

$$S=S_{i_1}^{\lambda_1}S_{i_2}^{\lambda_2}\cdots S_{i_k}^{\lambda_k},$$

where the exponents λ_j are integers. We call the sum

$$m=\sum_{j=1}^{k}|\lambda_j|$$

the grade of S and that of the image $S(B_0)$ of B_0. In particular, the identical transformation and B_0 have the grade 0, and any generator S_i ($i=1, 2, \cdots, p$) together with its inverse S_i^{-1} and the image $S_i(B_0)$ of B_0 have the grade 1.

Consider an infinite set of circles which are obtained from p pairs of circles H_i, H_i' ($i=1, 2, \cdots, p$) of B_0 by all the transformations of G. We say that a circle of the set is of grade m, if it is surrounded by m circles of the set. The total number of circles of grade m is obviously equal to $2p(2p-1)^m$. If we perform a transformation of grade m (>0) on B_0, we obtain a domain of grade m whose outer boundary is a circle of grade $m-1$ and inner boundaries are $2p-1$ circles of grade m.

Denote by D_m a domain bounded by the whole circles of grade m. Then D_m ($m=0, 1, 2, \cdots$) are a monotone increasing sequence of domains, so that D_μ ($\mu<m$) is contained in D_m as a subdomain. Further, denote by D_m^c the complement of D_m with respect to the extended z-plane. Then D_m^c consists of $2p(2p-1)^m$ closed disks which are mutually disjoint. For $m\to\infty$ D_m^c converges to a perfect non-dense set E. We call E the singular set of G. G is properly discontinuous in the complement of E.

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