ON SOME EXAMPLES OF SEMIGROUPS

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By a semigroup is meant a set S of elements a, b, . . . closed under an associative binary operation:

$$(ab)c = a(bc).$$

We shall say that S is left regular, right regular, left simple or right simple if it satisfies the following condition respectively.

I. Left regularity: $ax = ay$ for some a implies $x = y$.
II. Right regularity: $xa = ya$ for some a implies $x = y$.
III. Left simplicity: For any a,b, there exists some x such that $xa = b$.
IV. Right simplicity: For any a,b, there exists some x such that $ax = b$.

The structure of semigroups S satisfying any two postulates taken from the above four is well-known in the following four cases.

(1) S is left regular and right regular,
(2) S is left regular and right simple,
(3) S is right regular and left simple,
(4) S is left simple and right simple.

In the first case (1), the structure theorem is seen in [1].

In the second case (2), S is represented as the following manner:

$$S \cong G \times R,$$

where G is a group, and R is a semigroup in which we have

$$ab = b \quad \text{for any} \quad a, b \in R.$$

This semigroup R is called right singular.

Dually in the third case (3), S is represented as the following manner:

$$S \cong G \times L,$$

where G is again a group, and L is a semigroup in which we have

$$ab = a \quad \text{for any} \quad a, b \in L.$$

This semigroup L is called left singular.

Remarks. Consider the following two postulates.

III'. For any a, b, the equation

$$xa = b$$

has a unique solution.

IV'. For any a, b, the equation

$$ax = b$$

has a unique solution.

It is easily velified that the postulate

III' implies II and III.

Dually we have the postulate

IV' implies I and IV.

Conversely the above-mentioned semigroup $G \times L$ satisfies the postulate III', i.e.,

II and III implies III'.

Dually we have

I and IV implies IV'.

Consequently,

III' is equivalent to II and III,