T. NISHIMOTO
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ASYMPTOTIC BEHAVIOR OF THE W-K-B APPROXIMATIONS NEAR A STOKES CURVE

By Toshihiko Nishimoto

§ 1. Introduction.

In this paper we consider the asymptotic behavior of solutions of the second order differential equation

$$\varepsilon^2\frac{d^2y}{dx^2}-p_0(x)y=0\,, \tag{1.1}$$

and of fourth order differential equations of the form

$$\varepsilon^2\frac{d^4y}{dx^4}-\left\{\left(p_0(x)+p_2(x)\varepsilon^2\right)\frac{d^2y}{dx^2}+\left(q_0(x)+q_2(x)\varepsilon^2\right)y\right\}=0 \tag{1.2}$$

for small positive parameter ε. The asymptotic analysis of the above equation (1.1) has been studied in connection with quantum mechanics by many authors and one equation of the form (1.2) is concerned with the Orr-Sommerfeld equation which appears in the stability theory of parallel flow of viscous fluids.

It is well known that the equation (1.1) and (1.2) have asymptotic solutions as $\varepsilon\to 0$ such that

$$\begin{aligned} &y_1(x)\sim p_0(x)^{-1/4}\exp\left\{\frac{1}{\varepsilon}\int^x\sqrt{p_0(x)}\,dx\right\}, \\ &y_2(x)\sim p_0(x)^{-1/4}\exp\left\{-\frac{1}{\varepsilon}\int^x\sqrt{p_0(x)}\,dx\right\}, \end{aligned} \tag{1.3}$$

and

$$y_1(x)\sim(x-a)u_1(x-a)\,,$$

$$y_2(x)\sim u_2(x-a)-\frac{q_0(a)}{p_0'(a)}y_1(x)\log(x-a)\,,$$

$$y_3(x)\sim p_0(x)^{-5/4}\exp\left\{\frac{1}{\varepsilon}\int^x\sqrt{p_0(x)}\,dx\right\},$$

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