H. KAMADA
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COMPACT EINSTEIN-WEYL FOUR-MANIFOLDS WITH COMPATIBLE ALMOST COMPLEX STRUCTURES

HIROYUKI KAMADA

1. Introduction

A Weyl manifold is a smooth conformal manifold (M, C) equipped with a torsion-free affine connection D preserving the conformal structure C. A Weyl manifold (M, C, D) is said to be Einstein-Weyl if its symmetrized Ricci tensor $r^{D(\mathrm{sym})}$ is proportional to a metric representative of C. The Levi-Civita connection ∇ of an Einstein manifold (M, g) gives an Einstein-Weyl structure $([g], \nabla)$ on M, where $[g]$ denotes the conformal structure determined by g. Thus the notion of Einstein-Weyl structures is a generalization of Einstein metrics, so there are many studies in this topic (see Pedersen-Swann [9], [10], Itoh [4], and their references).

An almost complex structure J on a conformal manifold (M, C) is said to be compatible if J preserves C. Let (M, C, J) be a conformal manifold with a compatible almost complex structure J. By making use of the Lee form β_g of each metric g in C, we can naturally define a unique Weyl connection D on (M, C, J), which is called the canonical Weyl connection associated with (C, J). In the 4-dimensional case, we shall call such a quadruple (M, C, D, J) an *almost Hermitian-Weyl* 4-manifold. It is known that for an almost Hermitian-Weyl 4-manifold (M, C, D, J), J is integrable if and only if J is parallel with respect to D. When J is D-parallel, (M, C, D, J) is called a Hermitian-Weyl manifold. Note that the definition of (almost) Hermitian-Weyl manifolds is very similar to that of (almost) Kähler manifolds. An almost Hermitian-Einstein-Weyl 4-manifold means an almost Hermitian-Weyl 4-manifold whose Weyl structure is Einstein-Weyl.

Sekigawa [6] showed that any compact almost Kähler-Einstein manifold with nonnegative scalar curvature must be Kähler-Einstein. Motivated by his result, we shall consider the integrability problem for almost Hermitian-Einstein-Weyl 4-manifolds. Our main result is as follows:

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