H. LI
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L^2 HARMONIC FORMS ON A COMPLETE STABLE HYPERSURFACES WITH CONSTANT MEAN CURVATURE*

HAIZHONG LI

Abstract

We show that an n-dimensional $(2 \leq n \leq 5)$ complete noncompact strongly stable hypersurface M with constant mean curvature in an $(n+1)$-dimensional manifold $\overline{M}$ of nonnegative bi-Ricci curvature admits no nontrivial L^2 harmonic 1-forms.

1. Introduction

Let $\overline{M}$ be an $(n+1)$-dimensional orientable Riemannian manifold and let $x : M \to \overline{M}$ be an immersion with constant mean curvature H of an n-dimensional differentiable manifold M into $\overline{M}$. We recall that x is *strongly stable* if (see [1], [2], [6])

$$I(f) \equiv \int_M \{|\nabla f|^2 - (|A|^2 + \overline{\mathrm{Ric}}(n))f^2\}\, dM \geq 0 \tag{1.1}$$

for all $f : M \to R$ with compact support, where ∇f is the gradient of f and $|A|^2$ is the squared norm of the second fundamental form of x, and $\overline{\mathrm{Ric}}(n)$ is the Ricci curvature of $\overline{M}$ in the unit normal direction n. We recall x is *weakly stable* (c.f. p. 127 of [2]) if (1.1) is true for all f with compact support that satisfies

$$\int_M f\, dM = 0. \tag{1.2}$$

In [3], do Carmo and Peng proved that if M is a strongly stable complete minimal hypersurface of an $(n+1)$-dimensional Euclidean space R^{n+1} with finite absolute curvature, then M is a hyperplane. In [1] and [2], Barbosa, do Carmo and Eschenburg proved that round spheres are the only compact hypersurfaces with constant mean curvature in R^{n+1} that are weakly stable. Mori [8] and da Silveira [4] considered the complete and noncompact surfaces with constant mean

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