W. MARZANTOWICZ
KODAI MATH. J.
20 (1997), 92—106

EQUIVARIANT CATEGORY OF THE FREE PART OF A G-MANIFOLD AND OF THE SPHERE OF SPHERICAL HARMONICS

WACŁAW MARZANTOWICZ*

Abstract

In this work we study the G-category of a G-manifold M by taking in consideration the fixed point set of a maximal torus of a compact Lie group G. The used method let us compute the G-category of sphere of every real irreducible, odd indexed representation V_l of the group $G=SO(3)$. An application to a nonlinear Dirichlet problem, one of several possible, is given. Simplifying a proof of estimate of the G-category of the free part of a sphere we also show that the complement of saturation of fixed point set of a maximal torus is an open invariant subset of larger G-category than the free part of action and give particular computation for the spherical harmonics.

0. Introduction

In study variational methods with symmetries it is very useful to apply invariant of mini-max type as genus of a G-space, G-category, or cohomological index of a G-space (see [Bar2] for a revue of recent results). In view of applications the most important is to know a value of such an invariant for the unit sphere of an orthogonal representation of a given compact Lie group G. It was first observed that if G is the torus $T=T^k$, or p-torus Z_p^k, p prime, then for every orthogonal representation V without fixed point of G on the sphere $S(V)$ a value of such an invariant for the sphere is equal to the complex dimension (or real dimension) of V (cf. [Fa], [C-P], [Ma1] and [Bar2] for other references). The situation changed drastically if the connected component G_0 of G is nonabelian (cf. [Bar1]). Using a method of classification of compact Lie groups with the Borsuk-Ulam property developed himself, T. Bartsch gave a condition on representation W_0 of $G=SO(3)$ (and an example) that for every other representation U of $SO(3)$, $U^G=0$, we have

$$\mathrm{cat}_G S(W_0 \oplus U) \leqq 2\, \mathrm{cat}_G S(W_0),$$

and consequently does not depend monotonic on the dimension ([Bar2]). On the

* Research supported by KBN grant 2-PO3A-080-08.
Received May 7, 1996; revised February 14, 1997.