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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF SECOND ORDER NONLINEAR DIFFERENCE EQUATIONS*

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Abstract

In this paper, we study the asymptotic behavior of the second order difference equation

$$(*)\qquad \Delta(r(n)\Delta x(n))+f(n, x(n))=0.$$

we obtain some sufficient conditions which ensure that all the solutions of $(*)$ are bounded, and also obtain some conditions which guarantee that for every solution $x(n)$ of $(*)$ satisfies $|x(n)|=O(R(n, n_0))$ as $n\to\infty$, where $R(n, s)=\sum_{k=s}^{n-1}\frac{1}{r(k)}$.

1. A discrete inequality

In the sequel we will require the following discrete inequality which extends the known discrete inequality obtained by Meng [5].

DEFINITION. A function $g(u)$ is said to belong to $\mathcal{F}$ if $g(u)$ is nondecreasing and continuous on $(0, \infty)$ and

$$g(u)/v\leqq g(u/v),\quad u\geqq 0,\ v\geqq 1.$$

Every where we mean that $\sum_{k=s}^{n}\alpha(k)=0$ if $n<s$.

LEMMA. *Let $x(n)$, $h_i(n)$, $i=1, 2, \cdots, m$ be real valued nonnegative functions defined on $N(n_0)=\{n_0, n_0+1, \cdots\}$, $n_0\in\{1, 2, \cdots\}$, $f(n)\geqq 1$ be nondecreasing on $N(n_0)$, $g_i(u)\in\mathcal{F}$, $i=1, 2, \cdots, m$. Suppose that the discrete inequality*

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