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SHARP ISOPERIMETRIC INEQUALITIES FOR STATIONARY VARIFOLDS AND AREA MINIMIZING FLAT CHAINS mod k

JAIGYOUNG CHOE*

It is well known [4, 5, 8] that a smooth minimal surface Σ spanning a rectifiable Jordan curve C satisfies the isoperimetric inequality

$$4\pi \text{ Area}(\Sigma) \leqq \text{Length}(C)^2, \tag{1}$$

where equality holds if and only if C is a circle and Σ is a disk. Some smooth minimal surfaces in $\boldsymbol{R}^3$ can be physically realized as soap films. However, the soap films formed by dipping some wire frames in soap solution contain interior singular curves. Here arises the main question of this paper: Does (1) still hold for this soap-film-like surface with singularities? In 1986 one of Almgren's results [2] answered this question affirmatively for area minimizing flat chains mod k. In this paper we extend his two-dimensional result and show that (1) holds also for two-dimensional stationary varifolds with connected boundary of multiplicity $\geqq 1$ (Theorem 2). Moreover, if the bounding curve C consists of k curves having the same end points, we obtain a new type of sharp isoperimetric inequality for area minimizing flat chains mod k spanned by C. Here, unlike (1), equality holds only for the union of k flat half disks with a common diameter (Theorem 3).

1. Arcs and sectors

In this section we derive sharp isoperimetric inequalities for domains in the plane where only a specific part of the boundary counts toward the length of the boundary.

LEMMA 1. *Let l_1 and l_2 be the rays emanating from a point O with an angle of $\theta \leqq \pi$. Let C be a curve from a point of l_1 to a point of l_2.*

(a) *Suppose that C lies in the smaller sector of the two formed by the rays (C may lie in either sector if $\theta = \pi$). Define D as the domain bounded by l_1, l_2, and C. Then*

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