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ON THE ZEROS OF FUNCTIONS WITH FINITE DIRICHLET INTEGRAL

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Abstract

We characterize subsets A of the unit disc in the plane such that every Blaschke sequence with elements in A is the sequence of zeros of an analytic function with finite Dirichlet integral.

Also, assuming that arguments are uniformly distributed independent random variables, we characterize the moduli of sequences which are almost surely zero sequences for the same function family.

1. Introduction

We consider the Dirichlet space $\mathcal{D}=\mathcal{D}(\Delta)$ of functions f analytic on $\Delta=\{z\in\boldsymbol{C}: |z|<1\}$ with finite Dirichlet integral

$$\|f\|^2=\int_\Delta |f'(z)|^2 dm . \tag{1}$$

By m we denote the planar measure on Δ. A simple computation shows that if $f(z)=\sum_{n=0}^\infty a_n z^n$ is analytic in Δ, then

$$\|f\|^2=\pi\sum_{n=0}^\infty n|a_n|^2 , \tag{2}$$

and so $\mathcal{D}$ is contained in the Hardy space H^2. In particular, this implies that every sequence $\{z_n\}$ of zeros (counting multiplicities) of a function $f\in\mathcal{D}$, $f\not\equiv 0$, (i.e. a zero sequence for $\mathcal{D}$) necessarily satisfies the Blaschke condition

$$\sum(1-|z_n|)<\infty . \tag{3}$$

It is well known that (3) and the condition

$$-\int_{-\pi}^{\pi}\log\operatorname{dist}(Z, e^{it})dt<\infty , \tag{4}$$

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