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ON THE JULIA DIRECTIONS OF THE VALUE DISTRIBUTION OF HOLOMORPHIC CURVES IN $P^n(C)$

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Abstract

The generalized Picard theorem [4] asserts that any non-constant holomorphic map f of C into $P^n(C)$ misses at most $2n$ hyperplanes in $P^n(C)$ in general position. In this paper we shall prove that for a transcendental holomorphic map f of C into $P^n(C)$ with an asymptotic value in $P^n(C)$, there exists a ray $J(\theta)=\{z=re^{\sqrt{-1}\theta}: 0<r<+\infty\}$ such that f, in any open sector with vertex $z=0$ containing the ray $J(\theta)$, misses at most $2n$ hyperplanes in $P^n(C)$ in general position.

1. Introduction

Let $f(z)$ be a non-constant meromorphic function on C. We regard f as a holomorphic curve in the Riemann sphere $P(C)$ by identifying $P(C)$ with $C\cup\{\infty\}$. Picard proved that f misses at most two values in $P(C)$. Using the theory of the normal family, G. Julia [5] proved the following result.

THEOREM A. *Let $f(z)$ be a transcendental entire function on C. Then there exists a ray $J(\theta)=\{z=re^{\sqrt{-1}\theta}: 0<r<+\infty\}$ such that f, in any open sector with vertex $z=0$ containing the ray $J(\theta)$, misses at most one value in C.*

H. Milloux [6] generalized Theorem A to meromorphic functions on C and proved the following result.

THEOREM B. *Let $f(z)$ be a transcendental meromorphic function on C with an asymptotic value in $P(C)$. Then there exists a ray $J(\theta)=\{z=re^{\sqrt{-1}\theta}: 0<r<+\infty\}$ such that f, in any open sector with vertex $z=0$ containing the ray $J(\theta)$, misses at most two values in $P(C)$.*

The ray $J(\theta)$ in Theorem A or Theorem B is called a Julia direction of f. Since a transcendental entire function always has an asymptotic value ∞ in $P(C)$, Theorem B is a generalization of Theorem A. We must note that not every transcendental meromorphic function has a Julia direction. In fact,

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