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CONSTRUCTION OF n-END CATENOIDS WITH PRESCRIBED FLUX

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1. Introduction

Let $X: \hat{\boldsymbol{C}}-\{q_1, \cdots, q_n\} \to \boldsymbol{R}^3$ be an *n-end catenoid*, that is, a complete minimal surface of genus 0 having n catenoid ends at q_i's, where $\hat{\boldsymbol{C}}:=\boldsymbol{C}\cup\{\infty\}$. Let $G: \hat{\boldsymbol{C}}-\{q_1, \cdots, q_n\} \to \boldsymbol{S}^2$ be its *Gauss map* which can be extended naturally on $\hat{\boldsymbol{C}}$, and let $w(q_i)$ denote the *weight* of the end q_i, that is, the similitude ratio of the asymptotic catenoid of the end q_i to the standard catenoid ($g=-z$, $\eta=-z^{-2}dz$). Remark that $w(q_i)$ takes negative value if the orientation of the end q_i differs from that of the standard catenoid, and that $w(q_i)$ vanishes if the end q_i is a flat end or is removed. The vector $w(q_i)G(q_i)$ is called the *flux vector* of the end q_i and, it follows from the *flux formula* (cf. e. g. [2]) that $\sum_{i=1}^n w(q_i)G(q_i)=0$. Now, conversely, we consider the following

PROBLEM. *Given n unit vectors $v_1, \cdots, v_n$ in $\boldsymbol{R}^3$ and n non-zero real numbers $a_1, \cdots, a_n$ satisfying $\sum_{i=1}^n a_i v_i=0$, is there an n-end catenoid $X: \hat{\boldsymbol{C}}-\{q_1, \cdots, q_n\} \to \boldsymbol{R}^3$ such that $G(q_i)=v_i$ and $w(q_i)=a_i$?*

In this paper, we study the problem in the case when q_i coincides with $\sigma(v_i)$ for each i, where $\sigma: \boldsymbol{S}^2 \to \hat{\boldsymbol{C}}$ is the stereographic projection from the north pole. Our main result is stated as follows.

THEOREM. *Let $v_1, \cdots, v_n$ be unit vectors in $\boldsymbol{R}^3$, and $a_1, \cdots, a_n$ non-zero real numbers satisfying $\sum_{i=1}^n a_i v_i=0$. Set $p_i:=\sigma(v_i)$ and*

$$F_i(z):=\frac{\bar{p}_i z+1}{z-p_i}.$$

Suppose there are complex numbers $b_1, \cdots, b_n$ satisfying

$$b_i \sum_{j\in N_i} b_j=a_i \qquad i=1, \cdots, n, \tag{1.1}$$

$$\sum_{j\in N_i} b_j F_i(p_j)=0 \qquad i=1, \cdots, n \tag{1.2}$$

and $\sum_{i=1}^n b_i\neq 0$, where $N_i:=\{j\in \boldsymbol{N} \mid 1\leqq j\leqq n,\ j\neq i\}$. Then there exists an n-end

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