S. ROPPONGI
KODAI MATH. J.
17 (1994), 214—227

THE HADAMARD VARIATION OF THE GROUND STATE VALUE OF SOME QUASI-LINEAR ELLIPTIC EQUATIONS

BY SUSUMU ROPPONGI

1. Introduction

Let Ω be a bounded domain in $\boldsymbol{R}^N$ ($N\geqq 2$) with smooth boundary $\partial\Omega$. Let $\rho(x)$ be a real smooth function on $\partial\Omega$ and ν_x be the exterior unit normal vector at $x\in\partial\Omega$. For any sufficiently small $\varepsilon\geqq 0$, let Ω_ε be the domain bounded by

$$\partial\Omega_\varepsilon=\{x+\varepsilon\rho(x)\nu_x\,;\ x\in\partial\Omega\}.$$

Fix $p\in(1, \infty)$ and let q be a fixed number satisfying $0<q<p^*-1$, where $p^*=\infty$ if $p\geqq N$ and $p^*=Np/(N-p)$ if $p<N$. Then we consider the following problem.

$$\lambda(\varepsilon)=\inf_{X_\varepsilon}\int_{\Omega_\varepsilon}|\nabla u|^p dx\,, \tag{1.1$_\varepsilon$}$$

where

$$X_\varepsilon=\{u\in W_0^{1,p}(\Omega_\varepsilon)\,;\ \|u\|_{L^{q+1}(\Omega_\varepsilon)}=1,\ u\geqq 0 \text{ a. e.}\}.$$

It is easy to see that there exists at least one non-negative solution u_ε which attains $(1.1)_\varepsilon$ and which satisfies

$$\begin{aligned} &-\operatorname{div}(|\nabla u_\varepsilon|^{p-2}\nabla u_\varepsilon(x))=\lambda(\varepsilon)u_\varepsilon^q(x) \qquad x\in\Omega_\varepsilon \\ &u_\varepsilon(x)=0 \qquad x\in\partial\Omega_\varepsilon \\ &u_\varepsilon(x)\geqq 0 \qquad \text{a. e. } x\in\Omega_\varepsilon. \end{aligned} \tag{1.2}$$

Furthermore $u_\varepsilon\in C^{1+\alpha}(\bar{\Omega}_\varepsilon)$ for some $\alpha\in(0, 1)$.

In this note we want to show the following.

THEOREM 1. *Assume that $p\geqq 2$ and $q\geqq p-1$. Assume that the minimizer u_0 of $(1.1)_0$ is unique. Then, the following asymptotic behaviour of $\lambda(\varepsilon)$ holds.*

$$\lambda(\varepsilon)-\lambda(0)=-\varepsilon(p-1)\int_{\partial\Omega}\left|\frac{\partial u_0}{\partial\nu_x}(x)\right|^p\rho(x)d\sigma_x+o(\varepsilon). \tag{1.3}$$

Here $\partial/\partial\nu_x$ denotes the derivative along the exterior normal direction.

Received February 9, 1993; revised September 20, 1993.