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ON THE VALUE DISTRIBUTION OF $f^l(f^{(k)})^n$

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Abstract

Quantitative estimations on the value distribution of $f^l(f^{(k)})^n$ are studied in this paper. As a result of this, some known results are improved.

1. Introduction

Let f denote a transcendental meromorphic function, and the usual symbols: $T(r, f)$, $N(r, f)$, $\bar{N}(r, f)$, $m(r, f)$, $S(r, f)$ of Nevanlinna value distribution theory see, e.g. [7], are used throughout the paper.

A complex value a is said to be a Picard value of f, if and only if, $f(z)-a$ has at most finitely many zeros. W. K. Hayman [8] conjectured that the only possible Picard value of $f^n f'$ is zero, and he himself proved the case when $n \geqq 3$ in [10], and left the cases of $n=1, 2$. Later on, Mues [11] proved the case for $n=2$, and afterwards Clunie [4] proved the case for $n=1$ when f is entire. An affirmative answer to the case when f is meromorphic and $n=1$ is yet to be resolved. Since then a stream of studies on questions of possible Picard values of differential polynomials of f has been launched, and many related results have been obtained, see e.g. [1]-[5] and [10]-[19]. In 1981, Steinmetz [12] proved:

THEOREM A. *Let f be a transcendental meromorphic function in the plane. If $n_0, \cdots, n_k \geqq 0$, $n_0 \geqq 2$, $n_1+\cdots+n_k \geqq 1$ and $\phi=f^{n_0}(f')^{n_1}\cdots(f^{(k)})^{n_k}-1$, then*

$$\limsup_{r\to\infty} \frac{\bar{N}(r, 1/\phi)}{T(r, \phi)} > 0.$$

In 1982, Doeringer [5] proved the following:

THEOREM B. *Let f be a transcendental meromorphic function, $Q(f)$ and $P(f)$ be two non-zero differential polynomials and $\phi=f^n Q(f)+P(f)$. Then for any natural number n with $n \geqq 3+\gamma_p(\gamma_p$: the weight of $P(f))$,*

$$\limsup_{r\to\infty} \frac{\bar{N}(r, 1/\phi)}{T(r, \phi)} > 0.$$

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