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RIEMANNIAN SUBMERSION WITH ISOMETRIC REFLECTIONS WITH RESPECT TO THE FIBERS

Dedicated to Professor Yoji Hatakeyama on his sixtieth birthday

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1. Introduction

The concept of Riemannian submersion was introduced by O'Neil [10] and is discussed by him and others ([4], [8], etc). A Riemannian submersion with totally geodesic fibers often appears in the differential geometry.

On the other hand, in [3] Chen and Vanhecke introduced the notion of the reflections with respect to submanifolds. And there are some studies of reflections with respect to the fibers in a Riemannian submersion or local fibering of a Sasakian manifold (e.g. [2], [9], [11]).

In this paper, we shall consider a Riemannian submersion $\pi: M \to N$ with fibers of dimension one. In Section 2, we give some properties of the integrability tensor A with respect to π. In Section 3, we shall consider the isometric reflections with respect to the fibers in Riemannian submersion which satisfies certain conditions. Our result is a generalization of the result of Kato and Motomiya [6], [11]. And particularly, in the case of 3-dimension, we get the following result: the reflections with respect to the fibers are isometries if and only if M admits a Sasakian locally ϕ-symmetric structure. Finally, we give a complete classification of 3-dimensional Riemannian manifolds with isometric reflections with respect to the fibers.

2. Riemannian submersion

In this section we collect some results on Riemannian submersions. Let $\pi: M \to N$ be a Riemannian submersion. Let X denote a tangent vector at $x \in M$. Then X decomposes as $\mathcal{V}X + \mathcal{H}X$, where $\mathcal{V}X$ is tangent to the fiber through x and $\mathcal{H}X$ is perpendicular to it. If $X = \mathcal{V}X$, X is called a vertical vector. If $X = \mathcal{H}X$, it is called horizontal. Let ∇ and $\tilde{\nabla}$ denote the Riemannian connections of M and N respectively.

We define tensors T and A associated with the submersion by

$$T_E F = \mathcal{V}\nabla_{\mathcal{V}E}\mathcal{H}F + \mathcal{H}\nabla_{\mathcal{V}E}\mathcal{V}F, \tag{1}$$

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