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KODAI MATH. J.
16 (1993), 379—397

THE LOGARITHMIC DERIVATIVE AND A HOMOGENEOUS DIFFERENTIAL POLYNOMIAL OF A MEROMORPHIC FUNCTION

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1. Introduction

In this note, by a meromorphic function we mean a function meromorphic in the complex plane $\boldsymbol{C}$. We shall here assume that the reader is familiar with the standard notation and terminology of value distribution theory (see for example, Hayman [1] or [3]). For a meromorphic function $g(z)$, which does not vanish identically, we can consider the logarithmic derivative $g'(z)/g(z)$. It plays an important role in Nevanlinna's theory of meromorphic functions. The following occupies the main part.

LEMMA. *Let $g(z)$ be a meromorphic function. If $g(z)$ is transcendental, we have*

$$m\left(r, \frac{g'}{g}\right)=O(\log^{+} T(r, g)+\log r) \tag{1.1}$$

as $r\to\infty$ through all values if $g(z)$ has finite order and as $r\to\infty$ outside a set of r of finite linear measure otherwise. If $g(z)$ is a rational function and not identically equal to zero,

$$m\left(r, \frac{g'}{g}\right)=o(1) \tag{1.2}$$

as $r\to\infty$ through all values.

For the sake of simplicity, we shall use the symbol "n. e. (nearly everywhere)" instead of tediously saying that possibly outside a set of r of finite linear measure.

W. K. Hayman pointed out the necessity of treating a homogeneous differential polynomial $g''g-2g'^{2}$ of an entire function $g(z)$ in his famous book [1; § 3.6, p. 77]. Concerning this proposal E. Mues [4] studied an influence of the zeros of $g''g-ag'^{2}$ with a complex number a on the entire function $g(z)$ itself.

Received April 28, 1992.