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SURFACES OF FINITE TYPE WITH CONSTANT MEAN CURVATURE

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Abstract

Which surfaces in the Euclidean space E^3 with constant mean curvature are of finite type? We show that a 3-type surface has non constant mean curvature. Moreover, among surfaces of revolution with constant mean curvature the only ones which are of finite type are: the plane, the sphere, the catenoid and the circular cylinder.

1. Introduction.

Finite type submanifolds were introduced by B.-Y. Chen in [1]. These can be regarded as a generalization of minimal submanifolds. From the class of finite type submanifolds the 2-type are those that attracted the interest and it is a striking fact that almost all the results concerned with 2-type spherical submanifolds. In [6] it was proved that every 2-type hypersurface M of S^{n+1} has non-zero constant mean curvature in S^{n+1} and constant scalar curvature. On the other hand, it was proved ([3], [4]) that every 3-type spherical hypersurface has non-constant mean curvature. As far as we know nothing is known about 3-type hypersurfaces of a Euclidean space E^{n+1}, with constant mean curvature.

It is well known that the minimal surfaces, the ordinary spheres and the circular cylinders in the Euclidean space E^3, are at most of finite 2-type. Moreover all these have constant mean curvature.

As it is known there is an abundance of surfaces of constant mean curvature in the Euclidean space E^3. Among them are certain of the surfaces of revolution, called Delaunay surfaces. Moreover, Wente [9] demonstrated the existence of an immersed torus of constant mean curvature and Kapouleas [7] has shown that there also exist compact immersed surfaces with constant mean curvature of every genus $g \geqq 3$.

In this article, we ask the following geometric question:

"Which surfaces in E^3 with constant mean curvature are of finite type?"

After some preliminaries we prove the following two theorems, which answer partially the question.

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