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MARGOLIS HOMOLOGY AND MORAVA K-THEORY FOR COHOMOLOGY OF THE DIHEDRAL GROUP

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Abstract

In this paper, we note that the Margolis homology $H(H^*(BG; \boldsymbol{Z}/p), Q_n)$ relates deeply the Morava K-theory $K(n)^*(BG)$. In particular we compute $K(n)^*(BD)$ for the dihedral group D by using Atiyah-Hirzebruch spectral sequence.

§ 0. Introduction.

Let G be a finite group and $H^*(BG; \boldsymbol{Z}/p)$ be the cohomology of G with the coefficient $\boldsymbol{Z}/p$ for a prime number p. Since the restriction map to a sylow p-group S of G is injective, it is important to know the cohomology of p-groups. However it seems a very difficult problem to compute $H^*(BS; \boldsymbol{Z}/p)$ when S is a nonabelian p-group. In this paper we consider the case $p=2$. The smallest nonabelian 2-groups S have the order 2^3, which have two types D and Q; the dihedral and the quaternion groups. The cohomology $H^*(BG; \boldsymbol{Z}/p)$, $G=D, Q$ are determined by Atiyah, Evens respectively [A], [E].

In this paper we first study the Margolis homology $H(H^*(BD; \boldsymbol{Z}/2), Q_n)$ for the dihedral group D and next study Morava K-theory $K(n)^*(BD)$ where $K(n)^*(-)$ is the cohomology theory with the coefficient $K(n)^*=\boldsymbol{Z}/p[v_n, v_n^{-1}]$. Such $K(n)^*(BD)$ are given by Tezuka—Yagita [T-Y2] using BP-theory. However we use here only Atiyah—Hizebruch spectral sequence for $K(n)^*$ theory. In particular we correct some inaccuracy of results in Tezuka—Yagita [T-Y2].

Quite recently I. J. Leary decided the muliplicative structure of $H^*(BG; \boldsymbol{Z}/p)$ for groups of order p^3 [Ly2] by using the cohomology of group $\tilde{G}$ which is the central product of G and 1-dimensional sphere S^1. The cohomology ring $H^*(BD; \boldsymbol{Z}/2)$ is very easy. But its Margolis homology seems not so easy. Hence we first study Margolis homology of $H^*(B\tilde{D}; \boldsymbol{Z}/2)$ and next consider that of $H^*(BD; \boldsymbol{Z}/2)$. I thank Nobuaki Yagita who introduced me to these problems.

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