C. O. KISELMAN
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REGULARITY CLASSES FOR OPERATIONS IN CONVEXITY THEORY

BY CHRISTER O. KISELMAN

Resumo

Glatecaj klasoj por operacioj en la teorio pri konvekseco. Ni enkondukas klasojn de glateco kiuj estas adaptitaj al la plej gravaj operacioj en la teorio pri konvekseco. Plej ofte ili estas inter la klasoj C^1 kaj C^2.

Abstract

We introduce regularity classes which are adapted to the most important operations in convexity theory. They are typically between C^1 and C^2.

1. Introduction.

The convex hull of a smoothly bounded set in $\boldsymbol{R}^n$ does not necessarily have a boundary of class C^2. This elementary observation is at the origin of the present paper. We ask what regularity such a convex hull must have, and we construct regularity classes which are adapted to the operation $A \mapsto \operatorname{cvx} A$ of taking the convex hull of a set in $\boldsymbol{R}^n$, as well as to other operations which are of interest in convexity theory: that of forming the vector sum $A+B$ of two sets A and B and that of projecting a convex set into a space of lower dimension. In all these cases, we reduce the question of regularity to that of a marginal function $f(x)=\inf_y F(x, y)$.

The simple example with A as the union of two disjoint Euclidean balls shows that the convex hull $\operatorname{cvx} A$ need not have C^2 boundary, but it is easy to see that the boundary in this case is of Hölder class $C^{1,1}$ (i.e., the boundary is described by a function whose derivative is Lipschitz continuous). Our regularity classes are generalizations of this Hölder class.

To describe the simplest case of our results, let A be a compact set in $\boldsymbol{R}^n$. If the boundary of A is of class $C^{1,1}$, then A is a union of Euclidean balls with radii bounded from below; if A is convex, the converse holds. The property of being such a union of balls is easily seen to be stable under the operation

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