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PROJECTIVE SPACES IN A WIDER SENSE, I

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Introduction.

The purpose of this paper is to generalize the notion of projective spaces in compact symmetric spaces. For pairs $\{o, p\}$ of antipodal points in a compact symmetric space M, we attach to each p a pair of totally geodesic submanifolds $(M_+{}^o, M_-{}^o)$ (simply denoted by (M_+, M_-)) in M. The general theory of (M_+, M_-) has been developed by B. Y. Chen and T. Nagano and now it plays an important role as a new method in the global study of compact symmetric spaces (cf. [5], [6]). We generalize projective spaces in terms of (M_+, M_-).

The aim of our study was to find a geometry for exceptional Lie groups. The compact Lie group $F_{4(-52)}$ is realized as the isometry group of the Cayley projective plane and the non-compact Lie group $E_{6(-26)}$ is the projective transformation group. For the compact exceptional Lie groups E_6, E_7 and E_8, we would like to find good symmetric spaces which play the same role as the Cayley plane does. The study originates from H. Freudenthal [7] and B. A. Rozenfeld [10].

First we intended to solve a problem proposed by H. Freudenthal (p. 175, [7]). Roughly speaking, it asks us whether the adjoint compact symmetric spaces of type EIII, EVI and EVIII (in the sense of E. Cartan) can be regarded as generalized projective planes. This problem was solved affimatively in [2], [3] and [4].

We know a unified construction of real simple Lie algebras (cf. [1]). In order to study the above problem, we constructed the usual projective planes explicitly by making use of the unifield algebras. Then we encountered the symmetric spaces of type EIII, EVI and EVIII, and moreover we obtained the real and the complex Grassmann manifolds $G^R(4, 4n)^*$ and $G^C(2, 2n)$ (cf. Example 1.2). We found some common structures existing in these spaces (cf. Definition 1.1) and we called the symmetric spaces with such structures the projective spaces in a wider sense (cf. [3], [4]).

In this paper especially the projective planes in the wider sense are studied. For these planes we first establish a duality between points and lines (cf. Corollary 1.8) and also give the intersection number of two lines. We list the classification of the planes at the end of this paper.

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