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KODAI MATH. J.
15 (1992), 221–229

THE CHARACTERISTICS OF BMOA ON RIEMANN SURFACES

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In this paper we give a John-Nirenberg type theorem for BMOA on general open Riemann surfaces. Using Ba spaces we give a new characteristic for BMOA on Riemann surfaces in this paper too.

1. Introduction.

In [7], T.A. Metzger asked whether the John-Nirenberg theorem for BMOA on the unit disk is true on Riemann surfaces. We have given a positive answer for compact bordered Riemann surfaces in [4]. In this paper we will give a John-Nirenberg type theorem for BMOA on general open Riemann surfaces. Some new characteristics of BMOA on Riemann surfaces will be given in this paper too.

2. John-Nirenberg type theorem for BMOA on Riemann surfaces.

Let R be an open Riemann surface which possesses a Green's function, i.e., $R\notin O_G$. Let $G_R(w, a)$ be the Green's function of R with logarithmic singularity at $a\in R$. We firstly give an important lemma as follows:

LEMMA 2.1. *Let $R_1\subset R_2\subset\cdots\subset R_k\to R$ be an exhaustion of the Riemann surface R, where R_k are compact bordered Riemann surfaces $(1\leqq k<\infty)$. F is an analytic function on R. Let the least harmonic majorant of the subharmonic function $|F(w)|^p$ on R(or R_k) be $H(w)$ (or $H_k(w)$). Then*

$$H(w)=\sup_{k\geqq 1}H_k(w)=\lim_{k\to\infty}H_k(w).$$

If $F(w)$ has no harmonic majorant on R (or R_k) we denote $H(w)=\infty$ (or $H_k(w)=\infty$).

Proof. It is easy to verify that $\{H_k(w)\}$ is an increasing sequence. By Hanack theorem we get that $\lim_{k\to\infty}H_k(w)=H_0(w)$ is a harmonic function, or $H_0(w)$

* Project supported by the National Natural Science Foundation of China.
Received October 25, 1991.