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REMARKS ON EFFECTIVE CURVATURE

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1. Introduction

Let Ω be a domain in $R_{x,y}^{2}$ with a reflecting smooth boundary Γ. Suppose that Ω is a media through which wave propagates with a speed $c(x, y)$. Let s be the arc length of Γ measured along the curve from a fixed point on Γ, and n be the normal distance from Γ to the point in $\bar{\Omega}$ such that internal points of Ω correspond to $n>0$. Now we suppose the center of curvature is in Ω. Let $K_0(s)$ be the curvature of Γ at s. Let the speed of wave propagation be constant. Then along a concave part of Γ ($K_0(s)>0$), a high frequency wave well known by the name of whispering gallery wave can propagate. When the speed is variable, the role of the boundary curvature $K_0(s)$ should be replaced by the effective curvature $K(s)$.

Babich and Kirpichnikova [1] defined the effective curvature $K(s)$ by

$$K(s)=K_0(s)+c^{-1}(s, n)\partial_n c(s, n)|_{n=0}. \tag{1.1}$$

Let ω be the frequency of wave and L_ε be the boundary layer given by

$$L_\varepsilon=\{(s, n)\in\Omega\,;\, K(s)>\varepsilon>0,\ n\geqq 0\}$$

where n is sufficiently small.

They constructed a solution U which satisfies the following Helmholtz equation asymptotically as $\omega\to+\infty$

$$(\Delta_{x,y}+\omega^2 c^{-2}(x, y))U=0 \qquad \text{in } L_\varepsilon \tag{1.2}$$

with the Dirichlet boundary condition

$$U|_\Gamma=0 \tag{1.3}$$

such that the solution is concentrated near Γ in the sense that

$$U \longrightarrow 0 \qquad \text{exponentially as } n\to+\infty. \tag{1.4}$$

Let

$$Ai(x)=\int_0^\infty \cos(t^3/3+xt)\,dt \qquad (x\in R)$$

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