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EXTREMAL FUNCTIONS FOR BLOCH CONSTANTS

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1. Introduction.

Let $f(z)$ be an analytic function in the unit disc $\Delta=\{|z|<1\}$ with $f'(0)\neq 0$. We denote by $r_f(z)$ the radius of the largest schlicht disc with center at $f(z)$ in the Riemann image surface of $f:\Delta\to\mathbf{C}$ (the complex plane). We also denote by $\tilde{r}_f(z)$ the radius of the largest disc which is contained in $f(\Delta)\subset\mathbf{C}$. Define r_f and $\tilde{r}_f$ by

$$r_f=\sup_{z\in\Delta} r_f(z),$$

$$\tilde{r}_f=\sup_{z\in\Delta} \tilde{r}_f(z),$$

respectively. Let $\mathcal{A}$ be a family of all analytic functions f in Δ with $f'(0)\neq 0$ and $\mathcal{A}_0$ be a family of all analytic functions in Δ with $f'(z)\neq 0$, $z\in\Delta$. Let $\mathcal{S}$ be a family of all univalent analytic functions in Δ. Then the Bloch, Landau, locally univalent Bloch and univalent Bloch constants are defined respectively by

$$B=\inf_{f\in\mathcal{A}}\frac{r_f}{|f'(0)|},$$

$$\mathcal{L}=\inf_{f\in\mathcal{A}}\frac{\tilde{r}_f}{|f'(0)|},$$

$$B_0=\inf_{f\in\mathcal{A}_0}\frac{r_f}{|f'(0)|},$$

$$B_1=\inf_{f\in\mathcal{S}}\frac{r_f}{|f'(0)|}.$$

For terminologies our basic references are [3] and [4].

The purpose of the present article is to show the following;

THEOREM 1. *Let $f(z)$ be one of the extremal function for the Bloch, Landau, locally univalent Bloch and univalent Bloch constants. Then*

$$\left|\lim_{\varepsilon\to 0}\frac{1}{\pi}\int_{\varepsilon<|z|<1}\frac{\overline{f'(z)}}{z^2 f'(z)}dx\,dy\right|\leqq 1$$

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