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AN EXAMPLE OF AN OPEN RIEMANN SURFACE NOT UNIFORMLY LARGE WITH RESPECT TO GREEN'S FUNCTIONS

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§1. Introduction and the main result.

Let R be an open Riemann surface. To assure certain uniformness of R, we may impose the following conditions on R.

(G) Assuming that R admits Green's functions $g(\cdot, q; R)$ with the pole $q \in R$, there is a positive constant M such that $\{p \in R : g(p, q; R) > M\}$ is simply connected for every $q \in R$.

(H) Letting $d_R(\cdot, \cdot)$ be Poincaré's hyperbolic distance on R, there is a positive ε such that $\{p \in R : d_R(p, q) < \varepsilon\}$ is simply connected for every $q \in R$.

A surface satisfying (H) is called one with a positive injectivity radius and has several nice properties (cf. [4]). The condition (G) is recently considered in [3] and [7].

Remark 1. The condition (G) implies (H). In fact, let F_p be a Fuchsian group acting on $\{|z|<1\}$ and corresponding to a universal covering map π_p of $\{|z|<1\}$ to a Riemann surface R satisfying (G) such that $\pi_p(0)=p$ for arbitrarily given $p \in R$. Then since $g(\pi_p(z), p; R)=\sum_{f \in F_p} \log|1/f(z)|$, π_p is injective on the disk $\{z : \log|1/z| > M\}$, which has some hyperbolic radius depending only on M.

Remark 2. In case of finite surfaces, (G) is equivalent to (H). In fact, in this case each of (G) and (H) is equivalent to the condition for non-existence of punctures.

But in general, (H) does not necessarily implies (G). Actually, the purpose of this note is to show the following.

THEOREM. *There is a regular Riemann surface of Parreau-Widom type which satisfies* (*H*) *but not* (*G*).

Here for regular Riemann surfaces of Parreau-Widom type, see for example [5]. We will construct a family of Riemann surfaces satisfying (H) but not (G) in §2, and give a proof of Theorem in §3.

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