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AUTOMORPHISM GROUPS OF HYPERELLIPTIC RIEMANN SURFACES

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Hurwitz [3] proved that a Riemann surface of genus g has at most $84(g-1)$ automorphisms. If the surface is hyperelliptic this bound may be very much shortened.

In this paper we obtain all surfaces of genus $g>3$ with more than $8(g-1)$ automorphisms, and their corresponding automorphism groups. As a consequence of these results, all hyperelliptic Riemann surfaces with more than $8(g-1)$ automorphisms appear to be symmetric.

The surfaces of low genus were studied by Wiman [12] and A. and I. Kuribayashi [4, 5].

The methods of our study of Riemann surfaces involve the representation of compact Riemann surfaces as quotient spaces of Fuchsian groups [6]. A Fuchsian group is a discrete subgroup of the group of orientable isometries of the hyperbolic plane D. If the quotient space D/Γ, Γ being a Fuchsian group, is compact, then Γ has the following presentation:

$$\langle a_1, b_1, \cdots, a_g, b_g, x_1, \cdots, x_r \mid a_1b_1a_1^{-1}b_1^{-1} \cdots a_gb_ga_g^{-1}b_g^{-1}x_1 \cdots x_r = x_i^{m_i}=1\rangle.$$

Then we call $(g, [m_1, \cdots, m_r])$ the signature of Γ and g is the genus of D/Γ. The numbers m_i are called proper periods. When X is a surface of genus g, it may be expressed as D/Γ, Γ having signature $(g, [-])$.

If G is a group of automorphisms of the Riemann surface D/Γ, then G may be written as Γ'/Γ, where Γ' is another Fuchsian group. A Fuchsian group K with signature $(g, [m_1, \cdots, m_r])$ has associated an area

$$\text{Area}\,[K]=2\pi\Big(2g-2+\sum_{i=1}^{r}\Big(1-\frac{1}{m_i}\Big)\Big)=2\pi|K|,$$

and the order of $G=\Gamma'/\Gamma$ is $|G|=|\Gamma|/|\Gamma'|$.

Let now D/Γ be a Riemann surface of genus g, and $G=\Gamma'/\Gamma$ its group of automorphisms. If $|G|>8(g-1)$, then $|\Gamma'|<\dfrac{2(g-1)}{8(g-1)}=\dfrac{1}{4}$. We list the Fuchsian groups Γ' with $|\Gamma'|<\dfrac{1}{4}$.

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