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THE GROUP OF SELF-HOMOTOPY EQUIVALENCES OF S^2-BUNDLES OVER S^4, I

Dedicated to Professor Nobuo Shimada on his 60th birthday

BY KOHHEI YAMAGUCHI

0. Introduction.

The set $Eq(X)$ of homotopy classes of self-homotopy equivalences of a based space X forms a group under the composition of maps, and it is called the group of self-homotopy equivalences of X. The group $Eq(X)$ has been studied by several authors since the paper of W. D. Barcus and M. G. Barratt [1] appeared.

However, we have not yet obtained an effective method for calculating it except classical ones, and its structure also has not been clarified sufficiently. Furthermore, very little is known about this group even when X is a simply connected CW complex with three cells which is not a H-space. In particular, when X is a total space of S^m-bundles over S^n, the group $Eq(X)$ was already considred for $X=S^m\times S^n$ in [7], [8], [17], for a principal S^3-bundle over S^n in [9], [13], [16], and for the real and complex Stiefel manifolds $W_{n,2}$ and $V_{n,2}$ in [10]. Recently, S. Sasao studied the group $Eq(X)$ in [15] for the total space of S^m-bundles over S^n under the stable range, $3<m+1<n<2m-2$.

On the othe hand, it seems to be very difficult to investigate it under the unstable range. However, we would like to consider it when X is simply connected and the total space of S^m-bundles over S^n for a small pair of integers (m, n). Since X is simply connected, $n, m\geqq 2$ and the cases $(m, n)=(2, 2)$ or $(2, 3)$ were already considered by P. J. Kahn [7] and N. Sawashita [8].

Then the purpose of this paper is to study the group $Eq(X)$ for the case $(m, n)=(2, 4)$ and we will treat its application in the subsequent paper in [22].

1. Notations and Results.

All spaces have base points, and all maps and homotopies preserve base points throughout this note. We denote by $[X, Y]$ the set of based homotopy classes of maps from X to Y, and we will not distinguish between a map and its homotopy class. Let $Z\{x\}$ (resp. $Z_m\{x\}$) be the infinite cyclic group (resp. the cyclic group of order m) generated by the element x. Let RP^n (resp. CP^n)

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