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SOME EXAMPLES EXHIBITING THE PROCEDURES OF RENORMALIZATION AND GAUGE FIXING.

--SCHWINGER-DYSON EQUATIONS OF FIRST ORDER

BY ATSUSHI INOUE

Abstract

In this note, a 'definition' of the useful but notorious Feynman measure corresponding to bilinear Lagrangeans with 'singular' coefficients is given through functional derivative equations. Especially, the procedures of renormalization and gauge fixing are clarified at the equation level.

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§1. Introduction.

When a mathematician wants to study the non-relativistic quantum theory, he may find the Schrödinger equation which is considered as governing the dynamics of non-relativistic quantum mechanics. Without taking physics into account, he may study properties of the solutions of the Schrödinger equation. Moreover, if he is lucky, he may say something to physicists.

In spite of this, when he wants to study the quantum field theory (Q.F.T.), he may be a little bit confused because there exists no mathematically well defined equation.

From our point of view, the so-called Schwinger-Dyson (S-D) equation, derived by using the functional derivative, seems a candidate of the equation governing the dynamics of the Q.F.T., but in general, it contains infinities be-

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