N. INNAMI AND K. SHIGA
KODAI MATH. J.
8 (1985), 259—263

THE AXIOM OF GENERALIZED HYPERSPHERES IN RIEMANNIAN GEOMETRY

BY NOBUHIRO INNAMI AND KOICHI SHIGA

0. Introduction.

A characterization of spaces of constant curvature is the most classical and interesting subject in Riemannian geometry ([1], [2], [3], [9], [10], [11], [12], [13], [14], [15], etc.). Many axioms which characterize spaces of constant curvature have been found out. The axiom of n-planes by Cartan [3] and the axiom of n-spheres by Leung-Nomizu [11] are well-known. In the present note we shall establish the *axiom of generalized hyperspheres* and apply it to geodesic spheres and horospheres to obtain characterizations of spaces of constant curvature.

We prepare the notation for giving our axiom. Let M be a Riemannian manifold of dimension $m \geqq 3$ and N be a hypersurface (at least of class C^2) in M. Suppose that N has a unit normal vector field v. For a characterization of spaces of constant curvature we may assume without loss of generality that $\phi: N\times[0, \varepsilon)\to M$ given by $\phi(q, t):=\exp tv_q$ is diffeomorphic onto its image for some positive ε, because curvature properties are local ones. If $\phi_t(q):=\phi(q, t)$ for any $q\in N$ and for each t, then $N_t:=\phi_t(N)$ is a hypersurface in M for each t. The family $\{N_t;\ t\in[0, \varepsilon)\}$ will be called the *family of generalized hypersurfaces* associated with ϕ. Set $c_q(t):=\exp tv_q$ for each $q\in N$.

We now introduce the axiom.

Axiom of generalized hyperspheres. For every point $p\in M$ and every $(m-1)$-dimensional subspace T'_p of T_pM, there exists a hypersurface N through p such that $T_pN=T'_p$ and N_t is umbilical at $c_p(t)$ for each $t\in[0, \varepsilon)$.

In this axiom there are many choices of N, since the axiom does not require that N_t is umbilical at $c_q(t)$ for any point $q\neq p$ in N.

Then we shall prove

THEOREM 1. *Let M be a Riemannian manifold of dimension $m\geqq 3$. If M satisfies the axiom of generalized hyperspheres, then M is a space of constant curvature.*

Applying Theorem 1 to geodesic spheres, we shall obtain

COROLLARY 2. *Let M be a Riemannian manifold of dimension $m\geqq 3$. If all*

Received July 20, 1984