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ON THE GROWTH OF MEROMORPHIC FUNCTIONS OF ORDER LESS THAN 1/2, III

Dedicated to Professor Mitsuru Ozawa on his 60 th birthday

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Introduction.

This paper is concerned with one aspect of the Nevanlinna theory of meromorphic functions in the plane $\boldsymbol{C}$. We shall assume acquaintance with the standard terminology of the Nevanlinna theory

$$T(r, f),\quad m(r, a, f),\quad n(r, a, f),\quad N(r, a, f), \cdots .$$

If $f(z)$ is meromorphic, we define

$$M(r, f)=\sup_{|z|=r}|f(z)|, \qquad m^*(r, f)=\inf_{|z|=r}|f(z)|.$$

A nonconstant function $f(z)$ of finite order ρ is further classified as having *maximal, mean,* or *minimal type* according as

$$\limsup_{r\to\infty} T(r, f)/r^{\rho}$$

is infinite, positive, or zero, respectively.

Now, let ρ and δ be numbers with $0\leqq\rho<1/2$, $1-\cos\pi\rho<\delta\leqq 1$, and let $\mathfrak{m}_{\rho,\delta}$ be the set consisting of all meromorphic functions $f(z)$ of order ρ with the property that there is an $a\in\boldsymbol{C}$ satisfying $f(0)\neq a$ and

$$(1)\qquad N(r, \infty, f)<(1-\delta)N(r, a, f)+O(1) \qquad (r\to\infty).$$

The following result is well known.

THEOREM A. *Let $f(z)\in\mathfrak{m}_{\rho,\delta}$. Then given $\varepsilon>0$, there is a sequence of $r\to\infty$ such that*

$$(2)\qquad \log m^*(r, f)>\frac{\pi\rho}{\sin\pi\rho}(\cos\pi\rho-1+\delta)(1-\varepsilon)T(r, f).$$

This result was conjectured by Teichmüller [7], and Gol'dberg [4] obtained (2) in the weaker form: $\log m^*(r, f)>K\,T(r, f)$, where K is a positive constant. The determination of the exact value of K is due to Ostrowskii [6].

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