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HESSIAN QUARTIC FORMS AND THE BERGMAN METRIC

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§ 0. Introduction and notation. In [7], the "*curvature*" of the Carathéodory metric on a bounded domain in C^m is considered by using the generalized Hessian of this metric; it may be called the *Hessian-curvature*. Referring to this, we define Hessian quartic forms to an arbitrary hermitian metric. These Hessian quartic forms enable us to provide another proof for the following result of Wu [14; Lemmas 1 and 4]: *The holomorphic sectional curvature coincides with the maximum of the Gaussian curvatures to all local one-dimensional submanifolds that contact at the point in the direction under consideration* (Corollary 1.8).

Modifying the construction of the *n-th order Bergman metric* introduced in [6] (also see [5]), we define quantities $\mu_{0,n}$ ($n \in N$) as follows: We consider a certain linear functional on a specified subspace of square-integrable holomorphic m-forms on a m-dimensional complex manifold and define the quantity μ_n by the square of the operator norm of this functional (Proposition 3.7). We then set $\mu_{0,n} := \mu_n/\mu_0$. The quantity $\mu_{0,n}$ is a $[0, +\infty)$-valued function on the tangent bundle, and is biholomorphic invariant (Theorem 4.2). Especially $\mu_{0,1}$ is the usual *Bergman metric*, and $2(\mu_{0,1})^2 - \mu_{0,2}$ is the quartic form defining the holomorphic sectional curvature of the Bergman metric (Theorem 4.4).

Let $\lambda_{0,n}^z$ be the n-th order Bergman metric on a complex manifold, relative to a coordinate z, as introduced in [6]. Then the Hessian quartic form of the Bergman metric coincides with $2(\mu_{0,1})^2 - \lambda_{0,2}^z$ (Corollary 5.4). In general, $\lambda_{0,2}^z \geqq \mu_{0,2}$ with an explicit statement as to when equality holds (Proposition 5.5). Finally, we note that the quantity $\lambda_{0,2}^z$ does depend on the coordinate z, by examining a concrete example (Corollary 5.8). One should observe, however, that while the quantity $\lambda_{0,n}^z$ with $n \geqq 2$ is biholomorphic invariant in the weak sense mentioned

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