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FOURIER COEFFICIENTS OF GENERALIZED EISENSTEIN SERIES OF DEGREE TWO II

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§1. Statement of Results.

This is a continuation of Part I ([13]). We treat a generalization of the previous result and investigate a non-vanishing property of Fourier coefficients. We follow the notation of Part I throughout this paper.

Let $f\in M_k(\Gamma_1)$ be a normalized elliptic eigen modular form of weight k, and let $[f]\in M_k(\Gamma_2)$ be the generalized Eisenstein series of degree two attached to f in the sense of Langlands [8] and Klingen [4] (see (1.4) of Part I). Then $[f]$ is an eigen modular form of weight k satisfying $\Phi([f])=f$ where Φ is the Siegel operator (Proposition in §2 of Part I); conversely, by Kurokawa [5], $[f]$ is characterized by these properties. Let

$$[f](Z)=\sum_{T\geqq 0} a(T,\,[f])\mathrm{e}(\mathrm{tr}(TZ)) \tag{1.1}$$

be the Fourier expansion of $[f]$, where $\mathrm{e}(z)=\exp(2\pi iz)$, tr denotes the trace of matrices, and T runs over all symmetric positive semi-definite semi-integral matrices of size 2. We investigate these $a(T,\,[f])$.

Let

$$f(z)=\sum_{n=0}^{\infty} a(n)\mathrm{e}(nz) \tag{1.2}$$

be the Fourier expansion of f, hence $T(n)f=a(n)f$ for each $n\geqq 1$. For $|T|=0$ we have $a(T,\,[f])=a(n)$ if T is unimodularly equivalent to $\begin{pmatrix} n & 0 \\ 0 & 0\end{pmatrix}$, since $\Phi([f])=f$. For $T>0$, from $T(p)[f]=(1+p^{k-2})a(p)[f]$ (p a prime number) we have:

$$\begin{aligned} a(pT,\,[f])=&(1+p^{k-2})a(p)a(T,\,[f])-p^{2k-3}a(p^{-1}T,\,[f]) \\ &-p^{k-2}a\left(p^{-1}T\begin{bmatrix} 0 & p \\ 1 & 0\end{bmatrix},\,[f]\right)-p^{k-2}\sum_{j=0}^{p-1} a\left(p^{-1}T\begin{bmatrix} 1 & 0 \\ j & p\end{bmatrix},\,[f]\right) \end{aligned} \tag{1.3}$$

as in Maaß [11]. Here we understand that $a(*,\,[f])=0$ unless $*$ is semi-integral. The relation (1.3) enables us to write each $a(T,\,[f])$ as an explicit $\boldsymbol{Z}(f)$-linear combination of $a(T,\,[f])$ for primitive $T>0$, where $\boldsymbol{Z}(f)$ is the integer ring of the totally real number field $\boldsymbol{Q}(f)=\boldsymbol{Q}(a(n)\,|\,n\geqq 1)$ and we say that $T=\begin{pmatrix} t_1 & t/2 \\ t/2 & t_2\end{pmatrix}$

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