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MANIFOLDS AND DISCRETE STRUCTURES

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Let M be a smooth manifold and let $C(M)$ be the algebra of smooth functions on M. It is well-known that the smooth structure of M is completely determined by the algebraic structure of $C(M)$, while the latter structure is determined by the behaviors of smooth functions restricted on an everywhere dense subset of M. Hence we might say that an everywhere dense subset of M has already sufficiently many informations on the smooth structure of M. This rather perspective view seems to give a plausible reason to the validity of the following fact. Let $\{a_n\}$ and $\{b_n\}$ $(n=1, 2, \cdots)$ be two sequences on M, each of which forms an everywhere dense subset of M. Then there is a diffeomorphism φ of M such that $\varphi\left(\bigcup_{n=1}^{\infty} a_n\right)=\bigcup_{n=1}^{\infty} b_n$. Actually, this will be proved without difficulty (Section 3). We note that in this case we have $\varphi(a_n)=b_{\sigma(n)}$, where σ is a bijective map of the set of positive integers.

In order to obtain more strict relation between diffeomorphisms and everywhere dense sequences on M, it is natural to ask under what condition there exists a diffeomorphism φ with $\varphi(a_n)=b_n$ $(n=1, 2, \cdots)$. This problem-setting will be approved if we consider that any manifold M is obtained by first taking a sequence $a_1, a_2, \cdots$ successively so as to make a dense set and then doing completion of this set. Thus, $\{a_n\}$ is, in a sense, regarded as a generating set of the manifold. Then the above problem implies that, if we have two generating sets $\{a_n\}$ and $\{b_n\}$ of M, under what condition we can find such a diffeomorphism φ of M, that keeps the orders of these generating sets. Really, it seems to be very difficult to approach this problem in general. However, we hope that the problem may turn our attention to various aspects of manifolds which relate continuous or smooth structures with discrete structures.

Besides, we like to make a remark that, in case M is compact, to give an everywhere dense sequence $\{a_n\}$ on M allows us to regard M as a compactification of the set of positive integers via the correspondence of n to a_n $(n=1, 2, \cdots)$. Hence in such a way the set of positive integers will be able to acquire a kind of notion on density through the geometric structure of M.

We say that two sequences $\{a_n\}$ and $\{b_n\}$, everywhere dense on M, define the same discrete structure on M, if there are a diffeomorphism φ on M and an integer n_0 such that $\varphi(a_n)=b_n$ for $n\geqq n_0$.

In the present paper, we first want to clarify a fact that there exists an in-

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