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WHEN DOES INVERSION PRESERVE CONVEXITY ?

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Dedicated to Professor Kentaro Yano on his seventieth birthday.

The study of the problem of fluid flow past a concave body in [2, 5] raised the question of determining which circles invert a given ellipse into a convex curve. This question was answered by one of the authors in [6] and our purpose here is to extend this work to more general convex curves and to consider the same question for the surface of a convex body in Euclidean space.

I. The problem in curve theory

As is often done in the theory of plane curves we take the unit normal N to the curve to advance the unit tangent T by $\pi/2$. Then in the Frenet equation $dT/ds=\kappa N$, s arc length, the curvature κ may be positive, negative or zero and reversal of the direction of travel along the curve changes the sign of κ.

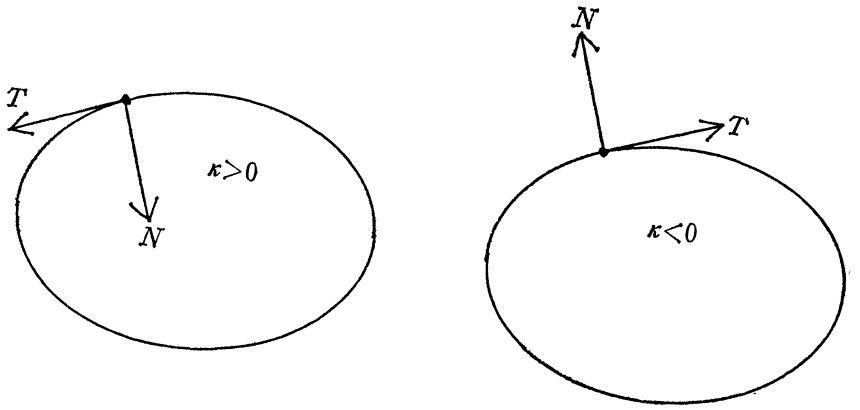

Fig. 1.

It is well known that a simple closed C^2 curve is convex if and only if κ has constant sign.

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