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OPERATIONAL CALCULUS OF TWO VARIABLES

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1. Introduction.

J. Mikusiński [2] has introduced a simple and complete operational calculus to obtain the solutions of linear ordinary differential equations with constant coefficients. The significance of operational calculus is that operators are regarded as convolution quotient, that is, functions and differential operator are elements of the same set. Moreover, he has discussed the linear partial differential equations of two variables with constant coefficients. In [2], the solutions were obtained by the traditional method of testing solutions by substituting the exponential functions, not by the systematic one as operational calculus. The difficulty of his method is that no prospective insight for obtaining the particular solutions has been given. In this paper, we discuss the linear partial differential equations of two variables with constant coefficients by the same systematic method as Mikusiński's operational calculus by imposing some restrictions on the constant part on the right hand side (cf. (2.7)).

We believe that the results obtained by such a way give a new development to the theory of Mikusiński's operational calculus.

In the last section, we show some well-known examples.

2. Operational calculus.

Let $\hat{k}$ be an algebrically closed field of characteristic 0 and $\mathcal{A}=\hat{k}[[\lambda]]$ be the module of the formal power series of a variable λ with coefficients in $\hat{k}$. Henceforth, we denote each element of $\mathcal{A}$ by $P(\lambda)$, or simply $P=\{\sum_{\nu=0}^{\infty} p_\nu \lambda^\nu\}$ (instead of usual notation $\sum_{\nu=0}^{\infty} p_\nu \lambda^\nu$) where $p_\nu \in \hat{k}$ ($\nu=0, 1, 2, \cdots$).

DEFINITION 2.1. Multiplication in $\mathcal{A}$ is defined by

$$(PQ)(\lambda)=\left\{\sum_{\rho=0}^{\infty}\left(\sum_{\rho=\nu+\mu} \frac{\nu!\mu!}{(\rho+1)!} p_\nu q_\mu\right)\lambda^{\rho+1}\right\} \tag{2.1}$$

where $P=\{\sum_{\nu=0}^{\infty} p_\nu \lambda^\nu\}$ and $Q=\{\sum_{\mu=0}^{\infty} q_\mu \lambda^\mu\}$.

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