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EXISTENCE OF QUASICONFORMAL MAPPINGS BETWEEN RIEMANNIAN MANIFOLDS

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Introduction.

In 1960 the first named author [8] proved that two Riemann surfaces are quasiconformally equivalent if and only if their Royden algebras are isomorphic. This result was extended to higher dimensions: to higher dimensional Euclidean domains by L. G. Lewis [6] and to Riemannian manifolds by J. Lelong-Ferrand [5]. These results show that if two Riemannian manifolds M and N are quasiconformally equivalent, then their Royden compactifications M^* and N^* are homeomorphic. The question aries whether the converse is true, that is, whether a homeomorphism from M^* to N^* can always be raised to a quasiconformal mapping from M to N.

In this paper we shall prove that the question is true in a neighborhood of ideal boundary of M, that is, if there is a homeomorphism f of M^* onto N^*, then there exists a compact subset K of M such that the restriction of f to each component of $M-K$ is quasiconformal. Furthermore, for Riemann surfaces, we can find a quasiconformal mapping from M to N. However we do not know whether this is valid for higher dimensional cases.

Notation and terminology

We denote by R^n the n-dimensional Euclidean space whose points x are n-tuple $x=(x_1, x_2, \cdots, x_n)$ of real numbers ($n\geqq 1$). The distance between $x=(x_1, \cdots, x_n)$ and $y=(y_1, \cdots, y_n)$ is denoted by

$$|x-y|=\left(\sum_{i=1}^{n}|x_i-y_i|^2\right)^{1/2}.$$

We denote by ω_{n-1} the $(n-1)$-dimensional Lebesgue measure of the unit sphere $\{x\in R^n;\ |x|=1\}$.

1. Riemannian manifolds

Let M be a connected separable, orientable n-idmensional ($n\geqq 2$) differentiable manifold of class C^1 with fundamental metric tensor

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