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ON FACTORIZATION OF ENTIRE FUNCTIONS

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1. Introduction. A meromorphic function $F(z)=f(g(z))$ is said to have f and g as left and right factors respectively, provided that f is meromorphic and g is entire (g may be meromorphic when f is rational). $F(z)$ is said to be prime (pseudo-prime, left-prime, right-prime) if every factorization of the above form into factors implies either f is linear or g is linear (either f is rational or g is a polynomial, f is linear whenever g is transcendental, g is linear whenever f is transcendental). When factors are restricted to entire functions, it is called to be a factorization in entire sense.

Gross [4] posed the following problem:

(A) Given any entire function f, does there exist a polynomial Q such that $f+Q$ is prime?

Further, Gross-Yang-Osgood [6] posed the following problem:

(B) Given any entire function f, does there exist an entire function g such that fg is prime?

In this paper we shall give affirmative answers to the above two problems (Theorem 2 and Theorem 3). Further we shall show a similar result for periodic entire functions (Theorem 4). In each case it can be shown that almost all functions are prime.

According to [9], [10], we shall make use of the simultaneous equations

$$\begin{cases} F(z)=c\,, \\ F'(z)=0\,. \end{cases}$$

Theorem 1 and Theorem 5 are extensions of theorem 1 and theorem 2 in [10].

2. In this section we shall state the following two theorems which are used in the proof of Theorem 2 and Theorem 3.

THEOREM A (a modified version of theorem 2 in [9]). *Let $F(z)$ be a transcendental entire function satisfying $N(r, 0, F')>km(r, F')$ on a set of r of infinite*

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