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KODAI MATH. J.
4 (1981), 251—265

CURVATURE INVARIANTS OF *CR*-MANIFOLDS

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§ 0. Introduction.

In [2], S. Bochner defined a certain curvature tensor as a complex analogy of Weyl conformal curvature tensor without geometrical interpretation. At present this tensor is called *Bochner curvature tensor*. Recently Webster [9], [11] gave this geometrical interpretation as a pseudoconformal invariant on a *CR*-manifold. Indeed Bochner curvature tensor is the 4th curvature invariant given in Chern-Moser's paper [4] (cf. Tanaka [7]). In this paper we shall also derive Bochner curvature tensor from our argument of *CR*-structure.

In [6] we studied almost contact structures standing on the viewpoint of pseudoconformal geometry and gave the change of canonical connections associated with almost contact structures belonging to the same *CR*-structure. The point under our discussion is the fact that almost contact structures belonging to a *CR*-structure play the same role as Riemannian structures belonging to a conformal structure and canonical connections correspond to Riemannian connections. Like the conformal change of Riemannian connections, a gradient vector appears in the change of canonical connections. Therefore we compute the difference of their curvature tensors and eliminate the gradient vector. Then we get a curvature invariant.

In § 1 we recall definitions and results given in [6]. § 2 is devoted to the study of curvatures of canonical connections. We in § 3 obtain the curvature invariant of the pseudo-conformal geometry.

The authors wish to express their hearty thanks to Professor S. Ishihara for his constant encouragement and valuable suggestions.

§ 1. Preliminaries.

Let $\mathcal{M}$ be a connected orientable C^∞-manifold of dimension $2n+1$ $(n\geqq 1)$ and $(\mathcal{D}, J)$ a pair of a hyperdistribution $\mathcal{D}$ and a complex structure J on $\mathcal{D}$. The pair $(\mathcal{D}, J)$ is called a *CR-structure* if the following two conditions hold:

(C. 1) $$[JX, JY]-[X, Y]\in\Gamma(\mathcal{D}),$$

(C. 2) $$[JX, JY]-[X, Y]-J([X, JY]+[JX, Y])=0$$

Received February 7, 1980